

(*mathematics*)  *inEconomics*

Mathematical Methods in Economics: Problems and Solutions

Chapter 7

Problems on Constrained Optimisation I

7.1 A firm produces two goods, X and Y , and has a total cost function of the form:

$$C = f(x, y) = 6x^2 + 3y^2$$

where C = total cost

x = quantity of good X produced (units)

y = quantity of good Y produced (units)

The firm wants to produce 21 items in total. How many of each should be produced to minimise cost? What is minimum cost?

7.2 A consumer has a utility function of the form:

$$U = u(x, y) = xy$$

where x = quantity of good X consumed (units)

y = quantity of good Y consumed (units)

The consumer's income to spend on X and Y is £20, and the consumer can buy X at a fixed price of £5 per unit and Y at a fixed price of £2 per unit. If the objective of the consumer is to maximise utility, how much of each good will be consumed? What is the marginal utility of money?

7.3 A consumer obtains utility from two goods X and Y and has the following utility function:

$$u = f(x, y) = xy$$

where u = index of utility

x = quantity of good X consumed (units)

y = quantity of good Y consumed (units)

The consumer has an income of £24.

(i) Find the marginal utility of X and the marginal utility of Y .

(ii) If the price of X is £6 per unit and the price of Y is £3 per unit, determine the quantities of X and Y that will maximise utility.

(iii) If the price of X falls to £4, and the price of Y increases to £4, determine the quantities of X and Y the consumer will now choose. Is the consumer better or worse off at the new set of prices?

7.4 A firm has a production function:

$$Q = f(L, K) = 12L + 20K - L^2 - 2K^2$$

where Q = output

L = labour input (units)

K = capital input (units)

The firm has a budget of £40 to spend. the price of labour is £4 per unit and that of capital £8 per unit. Determine the quantity of labour that the firm should use in order to maximise output. What is the maximum output?

7.5 A firm has 3 plants producing an identical commodity and it has a contract to supply 1,000 units of this commodity. The total cost functions for the three plants are:

$$\text{Plant 1: } C_1 = 100 + \frac{3}{100} q_1^2$$

$$\text{Plant 2: } C_2 = 100 + 2q_2 + \frac{1}{3,000} q_2^3$$

$$\text{Plant 3: } C_3 = 100 + 12q_3$$

where C_i = cost at plant i
 q_i = output (units) of plant i

How should output be allocated between the three plants so as to minimise the cost of producing 1,000 units?

7.6 A multiplant firm produces output in two different plants. The total cost function of the firm is:

$$C = c(q_1, q_2) = 0.1q_1^2 + 7q_1 + 15q_2 + 1,000$$

where C = total cost (£000's)
 q_1 = output of plant 1 (units)
 q_2 = output of plant 2 (units)

The firm has an order for 100 units. How much should the firm produce in each plant to minimise the total cost of production? What is the minimum total cost?

7.7 A consumer has a utility function:

$$u = f(q_1, q_2) = 4 \ln q_1 + 12 \ln q_2$$

where u = index of utility
 q_1 = quantity of good 1 consumed (units)
 q_2 = quantity of good 2 consumed (units)

- (i) Is the law of diminishing marginal utility operating here?
- (ii) If the consumer has an income of £120 and the price of good 1 is £3 per unit and that of good 2 is £5 per unit, how much of each good will be consumed? What is the marginal utility of money?

7.8 A firm has an order for 200 units of its product and wishes to distribute the manufacture of this order between two of its plants. The total cost function of the firm is:

$$C = f(q_1, q_2) = 2q_1^2 + q_1q_2 + q_2^2 + 200$$

where C = total cost
 q_1 = output of plant 1 (units)
 q_2 = output of plant 2 (units)

How much should the firm produce in each plant in order to minimise the cost of meeting the order? Show that when total cost is minimised marginal costs in each plant are equal.