(mathematics) √ inEconomics

WORKBOOK

Economic Applications 3

Contents

This workbook contains a set of problems that can be solved using mathematical techniques that include solving sets of linear equations containing up to three variables and using calculus to optimise a function of one variable. The functions used include polynomial (to degree two), algebraic and exponential.

Economic applications include perfectly competitive markets, tax incidence, the elasticity of demand and the elasticity of supply, total, average and marginal cost, total, average and marginal revenue, profit and revenue maximisation, break-even point, monopoly, the Keynesian model, the consumption function and the marginal propensity to consume and full employment income.

There are 23 problems together with worked solutions. Answers to these problems are given separately.

Page border from an edition of 'The King of the Golden River' by John Ruskin published as a pamphlet by James Brodie Limited. Illustrator and publication date unknown.

Mathematical Symbols and Letters from the Greek Alphabet used in Workbooks				
Mathematical symbols		***Letters from the Greek Alphabet***		
Symbol	**Meaning**	**Capital**	**Lower case**	**Name**
$\therefore$	therefore	Α	α	alpha
$\approx$	approximately equal to	Β	β	beta
$\equiv$	identically equal to	Δ	δ	delta
$<$	less than	Ε	ε	epsilon
$>$	greater than	Η	η	eta
$\leq$	less than or equal to	Θ	θ	theta
$\geq$	greater than or equal to	Λ	λ	lambda
$\Rightarrow$	implies	Μ	μ	mu
$\pm$	plus or minus	Ν	ν	nu
e	the exponential constant	Π	π	pi
∞	infinity	Ρ	ρ	rho
		$\sum$	σ	sigma

Using this workbook

The purpose of this workbook is to provide a set of problems that involve different aspects of economics and that can be solved using the mathematical techniques described on page 1. The problems are arranged randomly in terms of the techniques needed to solve them to help develop an ability to determine the appropriate technique needed to solve a problem. In general, problems towards the end of the workbook are more difficult to solve.

Worked solutions to the problems are given on pages 9-23. There is almost always more than one way of finding the solution to a problem. In most cases therefore, the worked solution given for a particular problem shows only one possible approach. There is no intention to suggest that the problem *has* to be solved using this approach or that it is necessarily a better approach than the alternatives. Where there is more than one problem with the same structure (for example two problems that involve solving the same type of equation) a different approach may be used to solve each one. Answers to all the problems are given on pages 24-27.

The best approach to using this workbook is to try solving some of the early problems and decide if they are appropriate to your level of mathematical ability. If they are too easy try some of the later problems, alternatively try the problems in the Workbook on Economic Applications 4 which includes some harder problems whose solutions can be found using the same set of mathematical techniques as used in this workbook. Solve a problem, check the solution, and then look at the answer given (pages 24-27). If your answer to the problem is not correct, and if *after some more work* you still cannot find the correct answer, look at the worked solution. There is nothing whatsoever to be gained by looking at the worked solution before a serious attempt has been made to try and solve the problem.

Since the problems in this workbook relate to different areas of economics some knowledge of the area of economics relevant to a particular problem is usually needed to solve it

Problems

1. A firm faces a fixed price of £8 per unit of output that it sells.

   (i) Find the total revenue, average revenue and marginal revenue function for this firm. To which class of functions do these three functions belong?

   (ii) Draw the total revenue curve. On another diagram draw the average and marginal revenue curves.

   (iii) What type of market structure could this firm be operating in?

2. A firm faces a demand function of the form:

   $$q = 20 - 0.5p$$

   where q = quantity demanded of the good the firm produces
   p = market price of the good the firm produces

   (i) Find the total revenue, average revenue and marginal revenue function for this firm. To which class of functions do these three functions belong?

   (ii) Compare the average and marginal revenue functions.

   (iii) Find the level of output at which total revenue is maximised.

   (iv) At what output levels is total revenue equal to zero? Explain why it is equal to zero at each of these levels of output.

   (v) Draw the total revenue curve and in a diagram below with the same scale measured on the horizontal axis draw the average and marginal revenue curves.

3. The table below gives the number of units of output that a firm can sell at different prices.

Price p	Quantity q
16	20
32	12
46	5
54	1

   (i) Find total revenue at each price for the firm.

   (ii) The total revenue (R) function takes the form: $R = f(q) = aq + bq^2$. Find the values of a and b and represent the total revenue function graphically.

   (iii) Find the level of output for which total revenue is maximised

   (iv) Find the marginal revenue function. What is marginal revenue at the revenue–maximising level of output?

Worked solutions

1. (i) Let R represent total revenue, A_R represent average revenue and p represent market price.

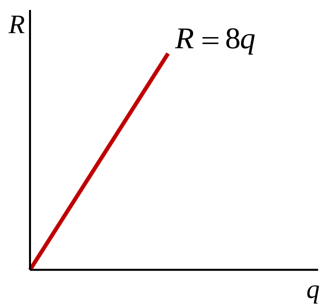
Total Revenue: $R = pq = 8q$

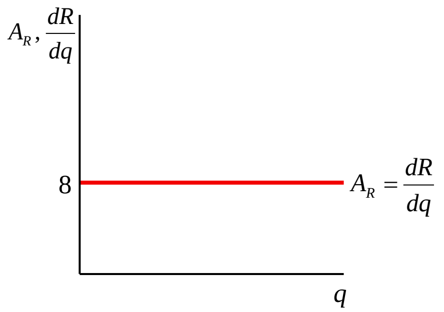
Average revenue: $A_R = \frac{R}{q} = \frac{8q}{q} = 8$ Average revenue is equal to price.

Marginal revenue: $\frac{dR}{dq} = 8$ Using the power function rule

These are all polynomial functions. The average and marginal revenue functions are constant functions (polynomials of degree zero) and the total revenue function is linear (polynomial of degree one). They are also rational functions and algebraic functions since all polynomial functions are also members of these two broader classes.

(ii)





(iii) This firm is facing a fixed price so it could be operating in a perfectly competitive market. Alternatively it could be operating in an oligopolistic market structure where price is set by the market leader.

2. (i) Total revenue is defined as $R = pq$ and the total revenue function expresses R in terms of q. In question 1 price takes a fixed value of 8 so substituting 8 for p gives an equation expressing R in terms of q. In this question since the firm faces not a fixed price but a downward sloping demand function, to express R in terms of q it is necessary to find p in terms of q and then substitute the expression for p in the definition of total revenue. To do this rearrange the demand function.

$$q = 20 - 0.5p$$

$$0.5p = 20 - q$$

$$p = 40 - 2q$$ This is called the *inverse* demand function.

Substituting this expression for p in the definition of total revenue:

$$R = pq = (40 - 2q)q = 40q - 2q^2$$

Average revenue: $A_R = \frac{R}{q} = \frac{pq}{q} = p = \frac{40q - 2q^2}{q} = 40 - 2q$ Average revenue equals price.

Marginal revenue: $\frac{dR}{dq} = 40 - 4q$

These are all polynomial functions. The average and marginal revenue functions are linear functions (polynomials of degree one) and the total revenue function is quadratic (polynomial of degree two). They are also rational functions and algebraic functions.

(ii)
$$A_R = 40 - 2q$$
$$\frac{dR}{dq} = 40 - 4q$$

These two functions have the same intercept on the vertical axis but their slopes differ. The slope of the marginal revenue function is twice as steep, $2(-2) = -4$, as that of the average revenue function.

(iii)
$$R = 40q - 2q^2$$

First-order condition for a stationary point:

$$\frac{dR}{dq} = 40 - 4q = 0 \qquad \Rightarrow \qquad q = 10$$

Second-order condition:

$$\frac{d^2R}{dq^2} = -4 < 0$$

Total revenue is maximised when $q = 10$.

(iv) If total revenue is equal to zero: $R = 40q - 2q^2 = 0$

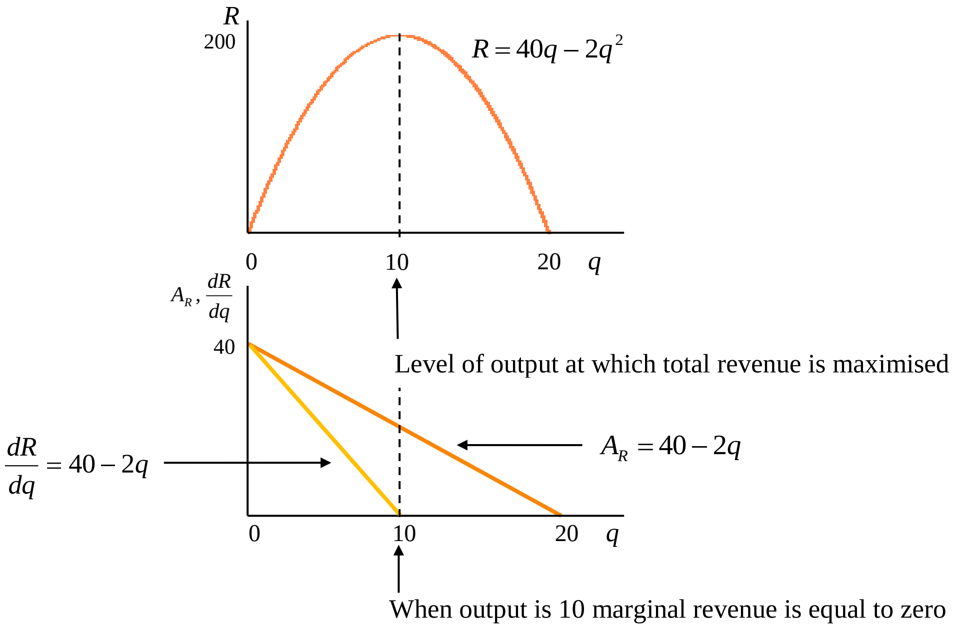
Solving this quadratic equation by factorisation:

$$R = q(40 - 2q) = 0 \qquad \Rightarrow \qquad q = 0 \quad \text{or} \quad q = 20$$

When $q = 0$ the firm is selling no output so $R = pq = p(0) = 0$.

When $q = 20$, from the average revenue function $p = 40 - 2q = 40 - 2(20) = 0$, the price at which output is sold is zero so $R = pq = (0)q = 0$.

(v)



3. (i)

Price p	Quantity q	Total revenue $R = pq$
16	20	320
32	12	384
46	5	230
54	1	54

(ii) All the pairs of values in (i) must satisfy the equation that defines the revenue function, $R = aq + bq^2$. Any two pairs will give a set of linear simultaneous equations in *a* and *b*.

$$320 = a(20) + b(20)^2$$

$$230 = a(5) + b(5)^2$$

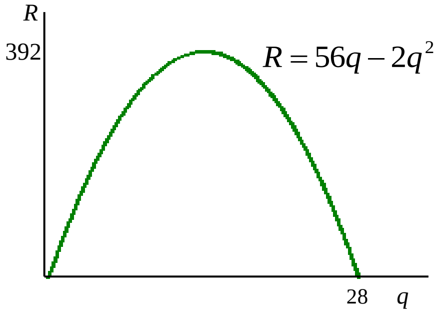
$$320 = a20 + b400 \qquad (1)$$

$$230 = a5 + b25 \qquad (2)$$

(2)×4 $\quad 920 = a20 + b100 \qquad (2a)$

$(1) - (2a) \quad -600 = b300 \quad \Rightarrow \quad b = -2$

Substituting $b = -2$ into (1): $\quad 320 = a20 + (-2)400 \quad \Rightarrow \quad a = 56$



(iii) $\quad R = 56q - 2q^2$

First-order condition for a stationary point:

$$\frac{dR}{dq} = 56 - 4q = 0 \quad \Rightarrow \quad q = 14$$

Second-order condition:

$$\frac{d^2R}{dq^2} = -4 < 0$$

Total revenue is maximised when $q = 14$.

(iv) Marginal revenue is given by $\frac{dR}{dq} = 56 - 4q$. When $q = 14$: $\frac{dR}{dq} = 56 - 4(14) = 0$