(mathematics) √inEconomics

WORKBOOK

Economic Applications 2

Contents

This workbook contains a set of problems that can be solved using some basic mathematical techniques like the problems in the workbook on Economic Applications I. The problems in this workbook are generally harder than the problems in that workbook. The mathematical techniques used include solving linear and non-linear equations, and sets of linear and non-linear equations containing up to three variables. No calculus is used. The functions that are used include polynomial (to degree three), rational, algebraic and exponential.

Economic applications include demand and supply, perfectly competitive markets, consumer and producer surplus, deadweight loss, lump-sum and proportionate taxation, stability, demand and supply of labour, tariffs, utility functions and indifference curves, total, average and marginal cost, net present value, the production possibility frontier and the Keynesian model including taxation and the budget deficit.

There are 21 problems together with worked solutions. Answers to these problems are given separately.

Page border from an edition of 'The King of the Golden River' by John Ruskin published as a pamphlet by James Brodie Limited. Illustrator and publication date unknown.

Mathematical Symbols and Letters from the Greek Alphabet used in Workbooks				
Mathematical symbols		***Letters from the Greek Alphabet***		
Symbol	**Meaning**	**Capital**	**Lower case**	**Name**
$\therefore$	therefore	A	α	alpha
$\approx$	approximately equal to	B	β	beta
$\equiv$	identically equal to	Δ	δ	delta
$<$	less than	E	ε	epsilon
$>$	greater than	H	η	eta
$\leq$	less than or equal to	Θ	θ	theta
$\geq$	greater than or equal to	Λ	λ	lambda
$\Rightarrow$	implies	M	μ	mu
$\pm$	plus or minus	N	ν	nu
e	the exponential constant	Π	π	pi
∞	infinity	P	ρ	rho
		$\sum$	σ	sigma

Using this workbook

The purpose of this workbook is to provide a set of problems that involve different aspects of economics and that can be solved using some basic mathematical techniques (see page 1). The problems are arranged randomly in terms of the techniques needed to solve them to help develop an ability to determine the appropriate technique needed to solve a problem. In general, problems towards the end of the workbook are more difficult to solve.

Worked solutions to the problems are given on pages 11-31. There is almost always more than one way of finding the solution to a problem. In most cases therefore, the worked solution given for a particular problem shows only one possible approach. There is no intention to suggest that the problem *has* to be solved using this approach or that it is necessarily a better approach than the alternatives. If there are two or more problems with the same structure (for example two problems that involve solving the same type of equation) the worked solution given may show a different approach to solving each problem so as to highlight the fact that there are different methods by which a solution can be found. Answers to all the problems are given on pages 32-35.

The best approach to using this workbook is to try solving some of the early problems and decide if they are appropriate to your level of mathematical ability. If they are too easy try some of the later problems. Solve a problem, check the solution, and then look at the answer (pages 32-35). If your answer to the problem is not correct, and if *after some more work* you still cannot find the correct answer, look at the worked solution. There is nothing whatsoever to be gained by looking at the worked solution before a serious attempt has been made to try and solve the problem.

Since the problems in this workbook relate to different areas of economics, some knowledge of the area of economics relevant to a particular problem is usually needed to solve it.

Problems

1. A firm has a total cost (C) function:

$$C = c(q) = \tfrac{3}{4}q^2 + \tfrac{8}{5}q + h$$

where q = output

(i) If the firm is producing 20 units of output average total cost is 21. What is the value of h and what does it represent?

(ii) What is average total cost when $q = 32$?

(iii) If total cost is 2,043 how much output is the firm producing?

2. An economy can be described by a simple Keynesian model. The equilibrium level of income in this economy is 455. The proportion of each extra unit of disposable income that is spent is 0.75, and currently there is a proportionate income tax of 20%. Firms spend $\bar{I}$. If the government reduces income tax to 10% find the change in the equilibrium level of income and the change in government revenue that will result.

3. (i) Represent the following functions graphically. p represents price and q represents quantity demanded.

$$p = f(q) = 12 - 2q$$

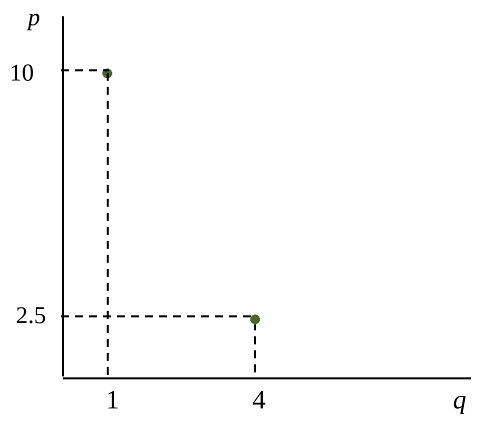
$$p = g(q) = \frac{10}{q}$$

$$p = h(q) = 2 + \frac{8}{q}$$

$$p = j(q) = \frac{72}{(q+1)^2}$$

$$p = m(q) = 4e^{2-q}$$

(ii) Which, if any, of the functions in (i) can explain the following observations?



Worked solutions

1. (i) Average total cost is given by;

$$\frac{C}{q} = \frac{\frac{3}{4}q^2 + \frac{8}{5}q + h}{q} = \frac{3}{4}q + \frac{8}{5} + \frac{h}{q}$$

When $q = 20$:

$$\frac{C}{q} = \frac{3}{4}q + \frac{8}{5} + \frac{h}{q} = \frac{3}{4}(20) + \frac{8}{5} + \frac{h}{20} = 21$$

$$15 + \frac{8}{5} + \frac{h}{20} = 21$$

$$\frac{h}{20} = 4.4$$

$$h = 88$$

h represents the firm's fixed costs.

(ii) When $q = 32$:

$$\frac{C}{q} = \frac{3}{4}q + \frac{8}{5} + \frac{88}{q} = \frac{3}{4}(32) + \frac{8}{5} + \frac{88}{32} = 28.35$$

(iii) When $C = 2{,}043$:

$$2{,}043 = \frac{3}{4}q^2 + \frac{8}{5}q + 88$$

$$0.75q^2 + 1.6q - 1{,}955 = 0$$

Solving this quadratic equation:

$$q = \frac{-1.6 \pm \sqrt{(1.6)^2 - 4(0.75)(-1{,}955)}}{2(0.75)} = \frac{-1.6 \pm 76.6}{1.5}$$

$$q = 50 \quad \text{or} \quad q = -52\tfrac{2}{15}$$

The firm is producing 50 units of output.

2. This model can be described as follows.

$$C = a + 0.75Y_d \qquad (1)$$

$$I = \bar{I} \qquad (2)$$

$$Y_d = Y - T \qquad (3)$$

$$T = 0.2Y \qquad (4)$$

$$Y = C + I \qquad (5)$$

The equilibrium level of income can be found by substituting for T from (4) in (3), substituting for Y_d in (1) then substituting for C and I in (5) to give:

$$Y = a + 0.75(Y - 0.2Y) + \bar{I}$$

$$Y - 0.6Y = a + \bar{I}$$

$$Y = \frac{a + \bar{I}}{0.4}$$

When $Y = 455$: $\quad 455 = \dfrac{a + \bar{I}}{0.4} \quad \Rightarrow \quad a + \bar{I} = 182$

If income tax is reduced to 10% the equilibrium level of income is given by:

$$Y = a + 0.75(Y - 0.1Y) + \bar{I}$$

$$Y - 0.675Y = a + \bar{I}$$

$$Y = \frac{182}{0.325} \qquad \text{Since } a + \bar{I} = 182$$

$$Y = 560$$

When $Y = 560$: $\quad T = tY = 0.1(560) = 56$

When $Y = 455$: $\quad T = tY = 0.2(455) = 91$

$$\Delta Y = 560 - 455 = 105$$

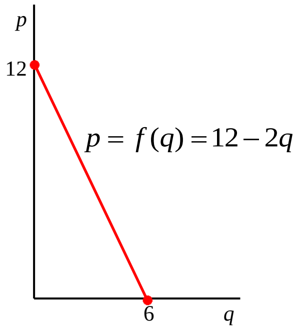
$$\Delta T = 56 - 91 = -35$$

3. To draw the graphs in this question the method used here is to find the coordinates of some points that lie on the graph, plot these points and join them up using a smooth curve. This is a very basic and inefficient approach and can only be applied with relatively simple functions because the graph of a complex function may have many features, turning points, asymptotes, discontinuities, etc., and even plotting many points may not be sufficient to identify all these features. A more efficient approach is to use a technique like calculus (used in subsequent workbooks) to identify important features and then draw the graph using this information. When drawing the graph of a function any information about features of the graph are helpful so it is useful to recognise the type of function and know the general shape of its graph. This will help minimise the number of points that need to be plotted to draw the graph. For example, the graph of a linear function is a straight line so it can be drawn by plotting any two points that lie on it, and joining them by a straight line. The graph of a quadratic function takes the shape of a parabola so has one turning point and is symmetric either side of it.

   Since these are all inverse demand functions, to be economically meaningful only the part of the graph where both p and q are non-negative is relevant, that is, the part of the graph that falls in the positive quadrant.

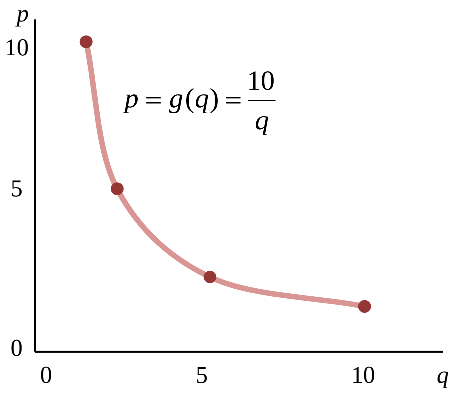
   (i) $p = f(q) = 12 - 2q$. This is a linear function and so its graph is a straight line and can be drawn by plotting any two points that lie on the graph (satisfy the equation that defines the function) and joining them by a straight line. Two points that it is convenient to plot are the point that lies on the *x*-axis and the point that lies on the *y*-axis, that is, the point where $p = 0$ and the point where $q = 0$.

q	0	6
p	12	0



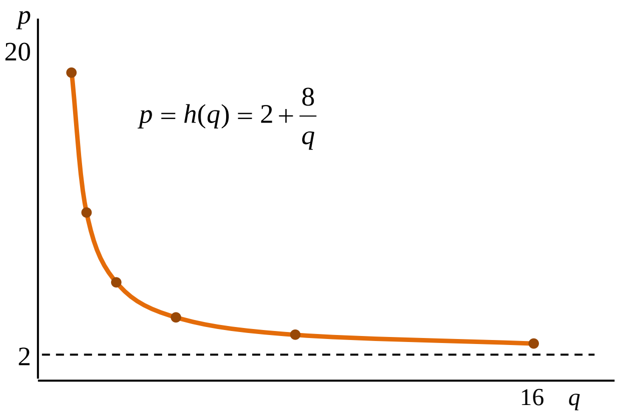
$p = g(q) = \frac{10}{q}$. This is a rational function. The graph of a rational function of this type is called a *rectangular hyperbola*. The *x* and *y* axes are asymptotes to the graph.

q	$q \to 0$	1	2	5	10	$q \to \infty$
p	$p \to \infty$	10	5	2	1	$p \to 0$



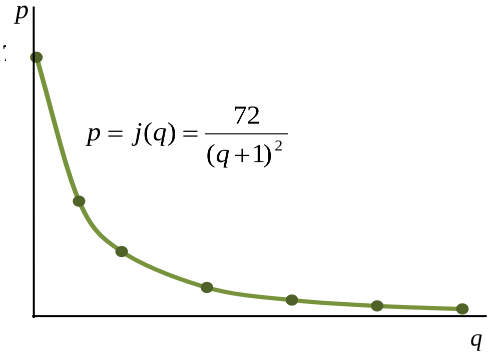
$p = h(q) = 2 + \frac{8}{q}$. This is a rational function.

q	$q \to 0$	0.5	1	2	4	8	16	$q \to \infty$
p	$p \to \infty$	18	10	6	4	3	2.5	$p \to 2$



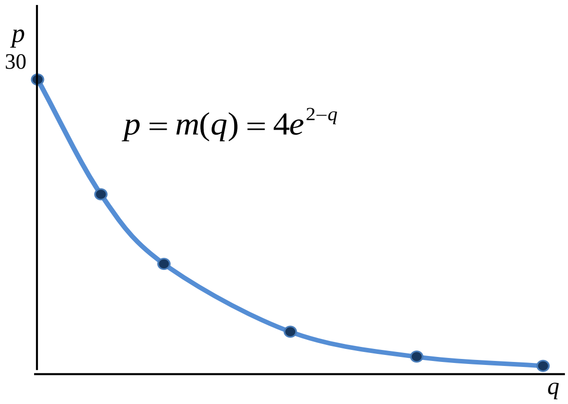
$p = j(q) = \frac{72}{(q+1)^2}$. This is a rational function.

q	0	0.5	1	2	3	4	5	$q \to \infty$
p	72	32	18	8	4.5	2.88	2	$p \to 0$



$p = m(q) = 4e^{2-q}$. This is an exponential function.

q	0	0.5	1	2	3	4	$q \to \infty$
p	$4e^2 \approx 29.56$	$4e^{1.5} \approx 17.93$	$4e^1 \approx 10.87$	$4e^0 = 4$	$4e^{-1} \approx 1.47$	$4e^{-2} \approx 0.54$	$p \to 0$



(ii)

q		
1	4	
$p = 12 - 2q = 12 - 2(1) = 10$	$p = 12 - 2q = 12 - 2(4) = 4$	
$p = \dfrac{10}{q} = \dfrac{10}{1} = 10$	$p = \dfrac{10}{q} = \dfrac{10}{4} = 2.5$	The plotted points lie on the graph of this function.
$p = 2 + \dfrac{8}{q} = 2 + \dfrac{8}{1} = 10$	$p = 2 + \dfrac{8}{q} = 2 + \dfrac{8}{4} = 4$	
$p = \dfrac{72}{(q+1)^2} = \dfrac{72}{(1+1)^2} = 18$	$p = \dfrac{72}{(q+1)^2} = \dfrac{72}{(4+1)^2} = 2.88$	
$p = 4e^{2-q} = 4e^1 \approx 10.87$	$p = 4e^{2-4} = 4e^{-2} \approx 0.54$	