(mathematics) √ inEconomics

WORKBOOK

Demand and Supply 2

Contents

The problems in this workbook, like those in the workbook Demand and Supply 1, are based on simple demand and supply models. However, in this workbook the problems are generally harder and more advanced mathematical techniques are needed to solve some of them. The mathematical techniques used include differentiating a function of two or more variables and integrating a function of one variable. Many of the functions used are non-linear and some are written in terms of parameters.

There are 23 problems together with worked solutions. Answers to the problems are given separately.

Page border from an edition of 'The King of the Golden River' by John Ruskin published as a pamphlet by James Brodie Limited. Illustrator and publication date unknown.

Mathematical symbols and letters from the Greek alphabet used in workbooks				
Mathematical symbols		***Letters from the Greek Alphabet***		
Symbol	**Meaning**	**Capital**	**Lower case**	**Name**
$\therefore$	therefore	A	α	alpha
$\approx$	approximately equal to	B	β	beta
$\equiv$	identically equal to	Δ	δ	delta
$<$	less than	E	ε	epsilon
$>$	greater than	H	η	eta
$\leq$	less than or equal to	Θ	θ	theta
$\geq$	greater than or equal to	Λ	λ	lambda
$\Rightarrow$	implies	M	μ	mu
$\pm$	plus or minus	N	ν	nu
e	the exponential constant	Π	π	pi
∞	infinity	P	ρ	rho
		$\sum$	σ	sigma

Using this workbook

This workbook contains a set of problems all of which relate to simple demand and supply models so knowledge of demand and supply analysis is needed to solve them.

Worked solutions to the problems are given on pages $12-45$. There is almost always more than one way of finding the solution to a problem. In most cases, therefore, the worked solution given for a particular problem shows only one possible approach to solving it. There is no intention to suggest that the problem *has* to be solved using this approach or that it is necessarily better than alternative approaches. If there are two or more problems with the same structure (for example two problems that involve solving the same type of equation) the method shown in the worked solution for solving one of them may be different from the method shown for another. The reason for this is to give additional information about how the problem can be solved. Answers to all the problems are given on pages $46-49$.

The problems in this workbook are arranged so that those for which more advanced mathematical techniques are needed to find the solution appear later in the workbook. A good approach to using this workbook is to find the solution to a problem, check it, and then look at the answer given. If your answer is not correct, and if *after some more work* you still cannot find the correct answer, look at the worked solution. There is nothing whatsoever to be gained by looking at the worked solution to a problem before a serious attempt has been made to try and solve it.

Problems

1. A perfectly competitive market is described by the following functions.

$$q_d = 102 - \tfrac{1}{2}p$$

$$q_s = -18 + \tfrac{3}{4}p$$

where q_d = quantity demanded

q_s = quantity supplied

p = price

(i) To reduce consumption of this good the government has decided to impose a lump-sum tax on consumers. What tax should it levy so as to also maximise government revenue?

(ii) By what percentage will consumption of this good fall if the tax in (i) is imposed?

(iii) What proportion of the tax is paid by consumers and what proportion by producers?

2. The demand function and supply function for a good sold in a perfectly competitive market are defined by the following equations.

$$q_d = 32 - 0.5p$$

$$q_s = -24 + 1.5p$$

where q_d = quantity demanded

q_s = quantity supplied

p = price

If, following the imposition by the government of a proportionate tax on producers, the equilibrium quantity traded in the market falls by 6 units what tax rate did the government impose?

3. The market demand function for a good takes the form:

$$q_d = 198 - 3p$$

where q_d = quantity demanded (units)

p = market price

The market supply function is linear and has a constant elasticity of unity. The market is currently in equilibrium and 33 units of the good are being traded at a price of 55 per unit.

(i) The government decides to impose an ad valorem tax on producers at a rate of $100t\%$ of the price of the good. Find the equilibrium market price and quantity traded following the imposition of the tax.

(ii) Represent this market graphically showing it before and after the imposition of the tax.

(iii) How will the equilibrium price and quantity traded change if the tax rate increases?

(iv) Does the incidence of the tax fall more heavily on consumers or producers?

Worked solutions

Note that sometimes to clarify the answer to a problem a graph that is not required as part of the answer is included in the worked solution.

1. (i) Let the lump-sum tax be represented by T. The quantity consumers demand depends on the price per unit that they pay so if a tax of T is imposed, the price they will pay is the market price plus the tax, that is, $p+T$. The demand function will therefore be given by:

$$q_d = 102 - \tfrac{1}{2}(p+T)$$

Revenue to the government is given by the tax paid on each unit multiplied by the number of units consumed. Assuming that this market is in equilibrium the number of units consumed is equal to the quantity traded at equilibrium so to find the value of T that will maximise government revenue the equilibrium quantity when a tax of T is levied on consumers is needed.

For equilibrium: $q_d = q_s$

$$102 - \tfrac{1}{2}(p+T) = -18 + \tfrac{3}{4}p$$

$$120 = \tfrac{1}{2}(p+T) + \tfrac{3}{4}p$$

$$p = 96 - \tfrac{2}{5}T$$

When $p = 96 - \tfrac{2}{5}T$, from the supply function:

$$q_s = -18 + \tfrac{3}{4}(96 - \tfrac{2}{5}T) = 54 - \tfrac{3}{10}T$$

Let p_e and q_e represent the equilibrium price and quantity traded in this market.

$$p_e = 96 - \tfrac{2}{5}T$$

$$q_e = 54 - \tfrac{3}{10}T$$

Let R_G represent revenue to the government.

$$R_G = Tq_e = T(54 - \tfrac{3}{10}T) = 54T - \tfrac{3}{10}T^2$$

First-order condition for a stationary point:

$$\frac{dR_G}{dT} = 54 - \tfrac{6}{10}T = 0 \qquad \Rightarrow \qquad T = 90$$

Second-order condition:

$$\frac{d^2R_G}{dT^2} = -\tfrac{6}{10} < 0$$

R_G is maximised when $T = 90$.

(ii) With no taxation: $q_e = 54 - \tfrac{3}{10}T = 54 - \tfrac{3}{10}(0) = 54$

When $T = 90$: $q_e = 54 - \tfrac{3}{10}T = 54 - \tfrac{3}{10}(90) = 27$

The percentage change in q_e is $\left(\frac{27-54}{54}\right)100 = -50\%$.

(iii) Tax to be paid per unit is 90.

With no taxation: $p_e = 96 - \frac{2}{5}T = 96 - \frac{2}{5}(0) = 96$

When $T = 90$: $p_e = 96 - \frac{2}{5}T = 96 - \frac{2}{5}(90) = 60$

So after the imposition of the tax consumers pay a price of 60 for each unit of the good which is 36 $(96-60)$ less than the price they paid before the tax was levied. They now, however, have to pay tax of 90 to the government so ultimately they pay 54 of the tax $(90-36)$. Producers pay the remainder, 36, through the reduction in the price that they receive per unit of the good.

Consumers pay: $\frac{54}{90} = 0.6$ (60%)

Producers pay: $\frac{36}{90} = 0.4$ (40%)

2. Before the imposition of the tax:

For equilibrium: $32 - 0.5p = -24 + 1.5p$

$$p = 28$$

Let q represent the equilibrium quantity traded.

When $p = 28$, using the demand function: $q = q_d = 32 - 0.5p = 32 - 0.5(28) = 18$

After the imposition of the tax the equilibrium quantity traded falls by 6 so the new equilibrium is $18 - 6 = 12$.

Let the proportionate tax rate be t. After the imposition of this tax producers will receive a price of $p - tp$ so the supply function can be written:

$$q_s = -24 + 1.5(p - tp)$$

For equilibrium: $32 - 0.5p = -24 + 1.5(1-t)p$

$$56 = (2 - 1.5t)p$$

$$p = \frac{56}{2 - 1.5t}$$

If a value for p can be found then this equation can be used to find the value for t. Since the equilibrium quantity after the imposition of the tax is 12, the equilibrium price can be found from the demand function. When $q = 12$:

$$12 = 32 - 0.5p \quad \Rightarrow \quad p = 40$$

When $p = 40$:

$$40 = \frac{56}{2 - 1.5t}$$

$$2 - 1.5t = \frac{56}{40}$$

$$t = 0.4$$

3. (i) To determine the equilibrium price and quantity traded in this market the supply function must be found.

The supply function is linear so it takes the following form where a and b are unknown constants.

$$q_s = a + bp$$

This function passes through the equilibrium point, that is, the point whose coordinates are $(55, 33)$. It has a constant elasticity of unity so it follows that:

$$\frac{dq_s}{dp}\frac{p}{q_s} = 1 \qquad \Rightarrow \qquad b\frac{55}{33} = 1 \qquad \Rightarrow \qquad b = \frac{3}{5}$$

Substituting $b = \frac{3}{5}$ and the coordinates of the equilibrium point into the supply equation gives:

$$33 = a + \tfrac{3}{5}(55) \qquad \Rightarrow \qquad a = 0$$

The market supply function takes the form:

$$q_s = \tfrac{3}{5}p$$

If a tax of $100t\%$ is imposed on suppliers the price per unit that suppliers will receive is $p - tp$ so the supply function will be:

$$q_s = \tfrac{3}{5}(p - tp) = \tfrac{3}{5}(1-t)p$$

The equilibrium price and quantity traded can now be determined.

For equilibrium: $q_d = q_s$

$$198 - 3p = \tfrac{3}{5}(1-t)p$$

$$198 = \tfrac{3}{5}(1-t)p + 3p$$

$$198 = (\tfrac{18}{5} - \tfrac{3}{5}t)p$$

$$p = \frac{330}{6-t}$$

Substituting $p = \frac{330}{6-t}$ in the demand function:

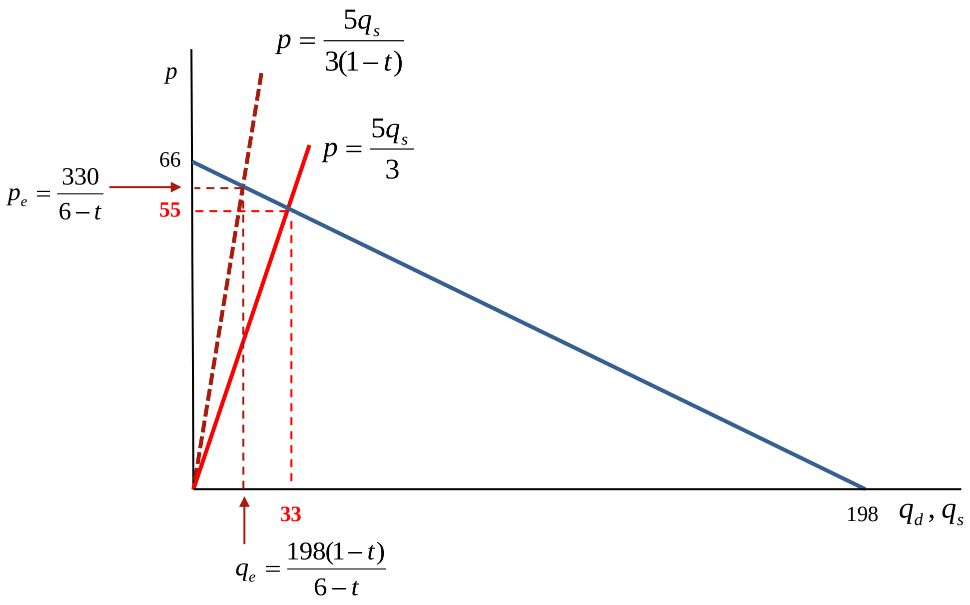
$$q_d = 198 - 3\left(\frac{330}{6-t}\right) = \frac{198(1-t)}{6-t}$$

The equilibrium price and quantity are:

$$p_e = \frac{330}{6-t}$$

$$q_e = \frac{198(1-t)}{6-t}$$

(ii)



(iii) $$\frac{dp_e}{dt} = (-1)(-1)\frac{330}{(6-t)^2} = \frac{330}{(6-t)^2} > 0$$ Using the chain rule

The equilibrium price will increase.

$$\frac{dq_e}{dt} = \frac{(6-t)(-198) - 198(1-t)(-1)}{(6-t)^2} = -\frac{990}{(6-t)^2} < 0$$ Using the quotient rule

The quantity traded will fall.

(iv) The total tax to pay per unit is $tp_e = t\left(\frac{330}{6-t}\right) = \frac{330t}{6-t}$

$$\Delta p_e = \frac{330}{6-t} - 55 = \frac{55t}{6-t}$$

The amount of the tax paid by consumers is therefore $\frac{55t}{6-t}$ since the market price has increased by this amount. The proportion of the tax paid by consumers is:

$$\frac{\left(\frac{55t}{6-t}\right)}{\left(\frac{330t}{6-t}\right)} = \frac{55}{330} = \frac{1}{6}$$

The remainder is paid by producers so they pay $\frac{5}{6}$ of the tax. The incidence of the tax therefore falls more heavily on producers.