(mathematics) inEconomics

Workbook

Demand and Supply 1

Contents

The problems in this workbook are based on simple demand and supply models. The mathematical techniques used in it include solving linear and quadratic equations, solving sets of linear equations and differentiating a function of one variable. The models are defined using linear functions only.

Aspects of demand and supply analysis covered in the problems include lump-sum and proportionate taxes, subsidies, tariffs, the incidence of a tax, price ceilings and floors, point elasticity of demand and supply, producer and consumer surplus, deadweight loss, a negative externality in production and revenue maximisation by a government or producers.

There are 22 problems together with worked solutions. Answers to the problems are given separately.

Page border from an edition of 'The King of the Golden River' by John Ruskin published as a pamphlet by James Brodie Limited. Illustrator and publication date unknown.

Mathematical Symbols and Letters from the Greek Alphabet used in Workbooks				
Mathematical symbols		***Letters from the Greek Alphabet***		
Symbol	**Meaning**	**Capital**	**Lower case**	**Name**
$\therefore$	therefore	A	α	alpha
$\approx$	approximately equal to	B	β	beta
$\equiv$	identically equal to	Δ	δ	delta
$<$	less than	E	ε	epsilon
$>$	greater than	H	η	eta
$\leq$	less than or equal to	Θ	θ	theta
$\geq$	greater than or equal to	Λ	λ	lambda
$\Rightarrow$	implies	M	μ	mu
$\pm$	plus or minus	N	ν	nu
e	the exponential constant	Π	π	pi
∞	infinity	P	ρ	rho
		$\sum$	σ	sigma

Using this workbook

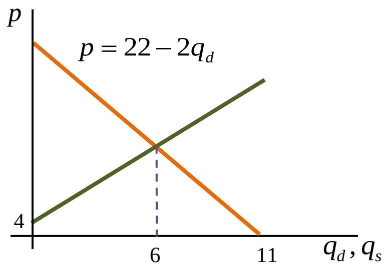
Since this workbook contains a set of problems that all relate to simple demand and supply models knowledge of demand and supply analysis is needed to solve them.

Worked solutions to the problems are given on pages $11-36$. There is almost always more than one way of finding the solution to a problem. In most cases, therefore, the worked solution given for a particular problem shows only one possible approach. There is no intention to suggest that the problem *has* to be solved using this approach or that it is necessarily better than alternative approaches. If there are two or more problems with the same structure (for example two problems that involve solving the same type of equation) the method shown in the worked solution for solving one of them may be different from the method shown for another. The reason for this is to give additional information about how the problem can be solved. Answers to all the problems are given on pages $37-42$.

The early problems in this workbook focus on the basic concepts of a demand and supply model so the best approach to using it is to start at the beginning and work through the problems in the order that they arranged in it. Solve a problem, check the solution, and then look at the answer given. If your answer to the problem is not correct, and if *after some more work* you still cannot find the correct answer, look at the worked solution. There is nothing whatsoever to be gained by looking at the worked solution before a serious attempt has been made to try and solve the problem.

Problems

1. A competitive market is described in the diagram below.



(i) What is the market equilibrium price?

(ii) What is the equation of the market supply function?

2. A perfectly competitive market is described by the following equations:

$$q_d = 48 - 2p$$

$$q_s = -22 + 3p$$

where q_d = quantity demanded
q_s = quantity supplied
p = market price

Find the market equilibrium price and quantity traded.

3. The inverse demand and supply functions for a perfectly competitive market are given below.

$$p = 96 - 4q_d$$

$$p = 36 + 2q_s$$

where q_d = quantity demanded
q_s = quantity supplied
p = market price

Find the price and quantity traded if the market is in equilibrium. Represent this market graphically. Mark the equilibrium price and quantity traded on the diagram.

Worked Solutions

Note that sometimes to clarify the answer to a problem a graph that is not required as part of the answer to it is included in the worked solution.

1. (i) The quantity traded at equilibrium is 6. The equilibrium price can be found using the inverse market demand function:

$$p = 22 - 2q_d = 22 - 2(6) = 10$$

(ii)The market supply function is linear and so takes the form:

$q_s = a + bp$ where a and b are unknown constants

The supply curve passes through the equilibrium point where $p = 10$ and $q_s = 6$. It also passes through the point where $p = 4$ and $q_s = 0$. *a* and *b* must therefore take values that satisfy the following equations.

$$6 = a + b(10) \qquad (1)$$

$$0 = a + b(4) \qquad (2)$$

From (2): $a = -4b$

Substituting $a = -4b$ into (1) gives: $6 = -4b + 10b \quad \Rightarrow \quad b = 1$

The equation of the market supply function is $q_s = -4 + p$

2. For equilibrium in this market $q_d = q_s$ so this model can be written in terms of the following equations.

$$q_d = 48 - 2p \qquad (1)$$

$$q_s = -22 + 3p \qquad (2)$$

(1) and (2): Behavioural equations

$$q_d = q_s \qquad (3)$$

(3): Equilibrium equation

This model consists of a set of three linear simultaneous equations in three variables, q_d, q_s and p. Since two of the variables q_d and q_s are already written in terms of the third variable, *p*, this set of equations can be reduced to one equation in the variable p, from which the value of p can be found, by substituting for q_d and q_s in terms of p into equation (3). This is an application of the *substitution* approach to solving a set of simultaneous equations.

Substituting for q_d and q_s from (1) and (2) in (3):

$$48 - 2p = -22 + 3p$$

$$70 = 5p$$

$$p = 14$$

The demand or the supply equation can now be used to find the equilibrium quantity. Whichever is chosen the remaining equation should be used to check that the solution has been calculated correctly.

From the demand equation: $q_d = 48 - 2p = 48 - 2(14) = 20$

Check using the supply equation: $q_s = -22 + 3p = -22 + 3(14) = 20$

Let p_e and q_e represent the equilibrium price and quantity traded.

$$p_e = 14$$

$$q_e = 20$$

3. This model can be described by the following set of equations.

$$p = 96 - 4q_d \qquad (1)$$

$$p = 36 + 2q_s \qquad (2)$$

$$q_d = q_s \qquad (3)$$

One way to solve this set of linear simultaneous equations is to substitute the expression for p from (1) in (2):

$$96 - 4q_d = 36 + 2q_s$$
$$60 - 4q_d = 2q_s$$
$$q_s = 30 - 2q_d \qquad (2a)$$

From (3) substitute q_d for q_s in $(2a)$:

$$q_d = 30 - 2q_d$$
$$q_d = 10$$

To find the equilibrium price substitute $q_d = 10$ in (1):

$$p = 96 - 4q_d = 96 - 4(10) = 56$$

Check by substituting $p = 56$ in (2):

$$56 = 36 + 2q_s$$
$$q_s = 10$$

Let p_e and q_e represent the equilibrium price and quantity traded.

$$p_e = 56$$
$$q_e = 10$$