(mathematics) √ inEconomics

PAMPHLET

Demand and Supply Analysis

Mathematical Symbols and Letters from the Greek Alphabet used in Pamphlets				
Mathematical symbols		***Letters from the Greek Alphabet***		
Symbol	**Meaning**	**Capital**	**Lower case**	**Name**
$\therefore$	therefore	A	α	alpha
$\approx$	approximately equal to	B	β	beta
$\equiv$	identically equal to	Δ	δ	delta
$<$	less than	E	ε	epsilon
$>$	greater than	H	η	eta
$\leq$	less than or equal to	Θ	θ	theta
$\geq$	greater than or equal to	Λ	λ	lambda
$\Rightarrow$	implies	M	μ	mu
$\pm$	plus or minus	N	ν	nu
e	the exponential constant	Π	π	pi
∞	infinity	P	ρ	rho
		$\sum$	σ	sigma

Using this Pamphlet

This pamphlet has been written for students taking a first year module in economic theory. Its aim is to describe some basic demand and supply models and to help develop a good understanding of them with a set of problems that are directly related to the models described. Worked solutions to the problems are included together with the answers that are given separately.

The best approach to using this pamphlet is to read the text and then attempt the problems. When tackling a problem try to find the solution, check your working, and then look at the answer given to see if your answer is correct. If your answer is not correct, and if *after some more work* you still cannot find the correct answer, look at the worked solution. There is nothing whatsoever to be gained by looking at the worked solution before a serious attempt has been made to try and solve the problem.

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Contents

This pamphlet describes some basic demand and supply models and explains the impact on a market of government intervention in the form of price ceilings and floors and taxes and subsidies. Consumer and producer surplus are used to determine the impact on welfare of such forms of intervention.

All the demand and supply models are linear. The mathematical techniques used include solving linear and quadratic equations and calculus (differentiation) applied to a function of one variable.

A set of problems, including some involving non-linear models, that require more advanced mathematical techniques to find the solution are given in the Workbook on Demand and Supply 2.

Page border from an edition of ‘The King of the Golden River’ by John Ruskin published as a pamphlet by James Brodie Limited. Illustrator and publication date unknown.

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1. INTRODUCTION

Demand and supply analysis is an area of economics that aims to explain how price is determined in a market. The models used in this type of analysis are called demand and supply models. A market is characterised by two groups of participants, consumers (households) who purchase the good (or service) that is traded in the market and producers (firms) who supply it. The interaction of these two groups in the market determines market price. A model to describe a market must therefore contain a description of the behaviour of each of these groups of participants.

2. THE DEMAND FUNCTION

The behaviour of consumers in a demand and supply model is described by a relationship called a **demand function**[1]. A demand function relates the quantity of a good demanded to the variables that determine this quantity. It is assumed in the theory of demand that the quantity demanded (q_d) of a good depends on its price (*p*), the prices of other goods that are complements or substitutes to it ($p_1, p_2, \ldots, p_n$), consumers' income (*m*), consumers' tastes (*t*) and other factors. Expressing this information mathematically a demand function can be written in the following general form where *z* is included to represent any other variable that might be relevant in a particular context.

$$q_d = f(p, p_1, \ldots, p_n, m, t, z)$$

The equation below describes a specific demand function in which the quantity demanded of a good depends on its own price, the price of one other good and consumers' income.

$$q_d = 180 - 0.5p + 0.16p_1 + 0.4m$$

The negative coefficient attached to *p* means that the quantity demanded and price of the good are *inversely related*. This reflects the law of demand which states that more of a good will be demanded at a lower price. The positive coefficients attached to the other two independent variables mean that quantity demanded is directly related to each of them so, for example, if consumers' income (*m*) increases consumers will demand more of the good at any price.

Since the objective of a demand and supply model is to explain how market price is determined it is the relationship between q_d and *p* that is of greatest significance in this context. To focus on this relationship all the other variables that determine quantity demanded are held constant. The phrase[2] used in economics to indicate that variables in a relationship are being held constant is *ceteris paribus*. A demand function relating q_d and *p* with all the other variables that determine q_d held constant can be written in the following general form.

$$q_d = f(p) \qquad \textit{ceteris paribus}$$

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[1] In the context of a demand and supply model it is a **market** demand function that is relevant. A market demand function gives the total quantity of a good that consumers who are buying in the market demand at any price. Market demand at a given price is equal to the sum of the quantities demanded by individual consumers at that price. Similarly a market supply function gives the total quantity that producers who are supplying the market are able and willing to supply at any price. Quantity supplied at a given price is equal to the sum of the quantities supplied by individual producers at that price.

[2] This is a Latin phrase meaning 'other things being equal'.

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The following equation is an example of such a function[3].

$$q_d = 30 - 0.25p \quad \textit{ceteris paribus}$$

The graph of the relationship between q_d and p is called a **demand curve**. There is an unusual feature to a demand curve. Despite the fact that in the algebraic representation of a demand function, $q_d = f(p)$, quantity demanded is the dependent variable and therefore, following the approach taken in mathematics should be measured on the vertical axis when graphing this relationship, the convention is to represent price on this axis[4] . This convention follows from the work of an economist called Alfred Marshall. The two-dimensional diagrams showing a demand curve and a supply curve that are used in economic analysis result from his work. His thinking was that in the relationship between quantity demanded and price, price was the dependent variable and so he measured price on the vertical axis when graphing this relationship. The reason that quantity demanded is the dependent variable in the algebraic representation of a demand function is because this representation developed from the work of other economists, notably Leon Walras, who believed quantity demanded to be the dependent variable in the relationship. So, as a consequence of the historical development of economic analysis the graph associated with a function written $q_d = f(p)$ is drawn with p measured on the vertical axis.

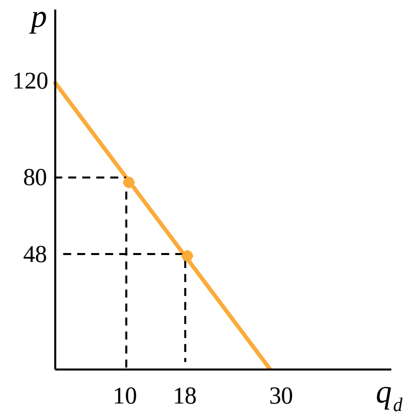
Associated with a demand function $q_d = f(p)$ is another function called the **inverse demand function**. This function expresses p in terms of q_d and can be written $p = f^{-1}(q_d)$. If mathematical conventions are followed a demand curve is actually the graph of the inverse demand function. For the demand function $q_d = 30 - 0.25p$ the inverse demand function can be found as follows.

$$q_d = 30 - 0.25p$$

$$q_d - 30 = -0.25p$$

$$p = 120 - 4q_d$$

The demand curve is shown in Figure 1.



A demand curve shows the quantity consumers will demand at each price. At a price of 80 consumers demand 10 units of the good and at a price of 48 they demand 18 units.

Figure 1

[3] Other ways in which a demand function may be represented are described in Appendix I to this pamphlet.

[4] This approach is used in most but not all textbooks in economics.

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Problems

1. The following equations define a perfectly competitive market.

$$q_d = 38 - 2p$$

$$q_s = -12 + 3p$$

where q_d = quantity demanded
q_s = quantity supplied
p = market price

(i) Find the equilibrium market price and the quantity that will be traded in the market at this price.

(ii) The price of a good that is a complementary to the good sold in this market falls resulting in a change in the quantity demanded of this good at each price. The new demand function is given by:

$$q_d = 48 - 2p$$

What is the percentage change in the equilibrium market price as a consequence of the change in demand?

(iii) Represent this market graphically showing the demand curve before and after the change in demand.

2. A perfectly competitive market is described by the following functions.

$$q_d = 50 - \tfrac{1}{2}p$$

$$q_s = -\tfrac{20}{3} + \tfrac{1}{3}p$$

where q_d = quantity demanded
q_s = quantity supplied
p = market price

(i) Find the market equilibrium price and quantity traded. Represent this model graphically.

(ii) Suppose that consumers' tastes change resulting in a new demand function of the form:

$$q_d = 72 - \tfrac{2}{3}p$$

At the same time as a consequence of an increase in the price of an input used in production of this good a new supply function of the following form results:

$$q_s = -8 + \tfrac{2}{7}p$$

Find the percentage change in the quantity traded in the market at equilibrium. Represent the market before and after the changes on the same diagram.

(iii) Suppose that the price of a substitute good falls resulting in a new demand function of the form:

$$q_d = 30 - 0.4p$$

At the same time the price of a good that producers can manufacture using the same factors as are needed to produce this good increases resulting in a new supply function of the form:

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$$q_s = -12 + 0.3p$$

Find the percentage change in the quantity traded in the market at equilibrium. Represent the market before and after the changes on the same diagram.

(iv) Suppose that consumers' incomes increase resulting in a new demand function of the form:

$$q_d = 80 - \tfrac{5}{8}p$$

At the same time technological improvements result in a new supply function of the form:

$$q_s = -5 + \tfrac{5}{12}p$$

Find the percentage change in the quantity traded in the market at equilibrium. Represent the market before and after the changes on the same diagram.

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