(mathematics) inEconomics

PAMPHLET

First-order Linear Difference Equations in Economics

Mathematical Symbols and Letters from the Greek Alphabet used in Pamphlets					
Mathematical symbols		***Letters from the Greek Alphabet***			
Symbol	**Meaning**	**Capital**	**Lower case**	**Name**	
$\therefore$	therefore	A	α	alpha	
$\approx$	approximately equal to	B	β	beta	
$\equiv$	identically equal to	Δ	δ	delta	
$<$	less than	E	ε	epsilon	
$>$	greater than	H	η	eta	
$\leq$	less than or equal to	Θ	θ	theta	
$\geq$	greater than or equal to	Λ	λ	lambda	
$\Rightarrow$	implies	M	μ	mu	
$\pm$	plus or minus	N	ν	nu	
e	the exponential constant	Π	π	pi	
∞	infinity	P	ρ	rho	
		$\sum$	σ	sigma	

Using this pamphlet

This pamphlet has been written for students taking a degree in economics or one containing a significant element of economics and who are studying difference equations in this context. Its aim is to show how first-order linear difference equations arise in economic models and how models involving these equations can be solved. To help develop a good understanding of the material a number of problems that are directly related to the models described are included together with their worked solutions. The answers to them are given separately.

The best approach to using this pamphlet is to read the text and then attempt the problems. When tackling a problem try to find the solution, check your working, and then look at the answer given to see if your answer is correct. If your answer to the problem is not correct, and if *after some more work* you still cannot find the correct answer, look at the worked solution. There is nothing whatsoever to be gained by looking at the worked solution before a serious attempt has been made to try and solve the problem.

Contents

This pamphlet describes the characteristics of first-order linear difference equations and shows how these equations can be solved. Two dynamic economic models, the cobweb model to which the greater focus is given and a Keynesian national income model are used as examples of economic applications that involve first-order difference equations.

Eleven problems directly related to the models described are included in this pamphlet together with worked solutions to them. The answers are given separately.

Page border from an edition of ‘The King of the Golden River’ by John Ruskin published as a pamphlet by James Brodie Limited. Illustrator and publication date unknown.

Introduction

One feature of an economic model is its relationship to time. Depending on this relationship an economic model is classified as either static or dynamic. A model is **static** if all the variables in it are assumed to relate to the *same* point in time. In this situation time does not feature explicitly in the model. An example of a static model is the simple demand and supply model. It is implicit in this model that the values of the variables in it, quantity demanded, quantity supplied and price all relate to the *same* point in time. Analysis of a static economic model involves finding the equilibrium values of the endogenous variables and then determining how these values change following an exogenous (external) change to the model, that is, a change in an exogenous variable or parameter. Since it involves comparing equilibrium values of the endogenous variables this type of analysis is called **comparative static analysis**. *In a static model it is implicitly assumed that following an exogenous change the endogenous variables adjust to their new equilibrium values.*

A model is **dynamic** when the values of the variables in it relate to *different* points in time. To encompass this feature time (t) must be included explicitly in the model[1]. A model can be formulated with time treated as a discrete variable, that is, $t = 0,1,2,\ldots$ or as a continuous variable, that is, $t \geq 0$. When time is treated as a discrete variable the model is called a discrete time model and analysis of it involves working with difference equations. When time is treated as a continuous variable the model is called a continuous time model and analysis of it involves using differential equations. Analysis of a dynamic model is called **dynamic analysis**. Dynamic analysis makes it possible to find the time path of the variables in an economic model and thus determine under what conditions, if at all, the values of these variables will converge on the new equilibrium values that follow from an exogenous change.

Analysis of a static model is described in the next section. Subsequent sections describe aspects of dynamic analysis.

Comparative static analysis

As described above, comparative static analysis is concerned with understanding what determines an equilibrium position *at a point* in time and how this equilibrium will change if there is an exogenous change to the model. As an example of a static model consider the following model of a perfectly competitive market.

		Example	
$q_d = \alpha - \beta p$	(1)	$q_d = 20 - 2p$	(1)
$q_s = -\gamma + \delta p$	(2)	$q_s = -4 + p$	(2)

where q_d = quantity demanded

q_s = quantity supplied

p = market price

α, β, γ and δ are positive constants

In addition to the relationships above an assumption of this model is that the market is in

[1] In this pamphlet time is introduced into a model by *dating* a variable, that is, by defining its value in a specific time period. This is necessary when dealing with models that involve relationships between the values of variables at different time periods for example between the quantity supplied in 2017 and market price in 2016. These are the types of relationships with which this pamphlet is concerned. Time, however, can also be introduced into a model by including it as an independent variable in a relationship that forms part of the model.

equilibrium when $q_d = q_s$. The implication of this equation is that market price adjusts to equate quantity demanded and quantity supplied, that is, it adjusts to *clear the market*. This equation is called the **equilibrium condition** and the price and quantity that result when it holds are called the **equilibrium price** and **equilibrium quantity**.

To solve this model, that is, to find the equilibrium price and quantity traded, substitute for q_d and q_s from (1) and (2) in the equilibrium condition to obtain an equation in *p* only that can be solved to give the equilibrium price.

$$\alpha - \beta p = -\gamma + \delta p \qquad\qquad 20 - 2p = -4 + p$$

$$p = \frac{\alpha + \gamma}{\beta + \delta} \qquad\qquad p = 8$$

Let p_e and q_e represent the equilibrium price and quantity traded. The equilibrium quantity traded can be found from the demand function, as shown below, or the supply function.

when $p_e = \dfrac{\alpha + \gamma}{\beta + \delta}$:

$$q_e = \alpha - \beta p_e = \alpha - \beta\left(\frac{\alpha + \gamma}{\beta + \delta}\right) = \frac{\alpha\delta - \beta\gamma}{\beta + \delta}$$

when $p_e = 8$:

$$q_e = 20 - 2p_e = 20 - 2(8) = 4$$

The market is in equilibrium when:

$$p_e = \frac{\alpha + \gamma}{\beta + \delta} \qquad\qquad p_e = 8$$

$$q_e = \frac{\alpha\delta - \beta\gamma}{\beta + \delta} \qquad\qquad q_e = 4$$

Representing the model graphically:

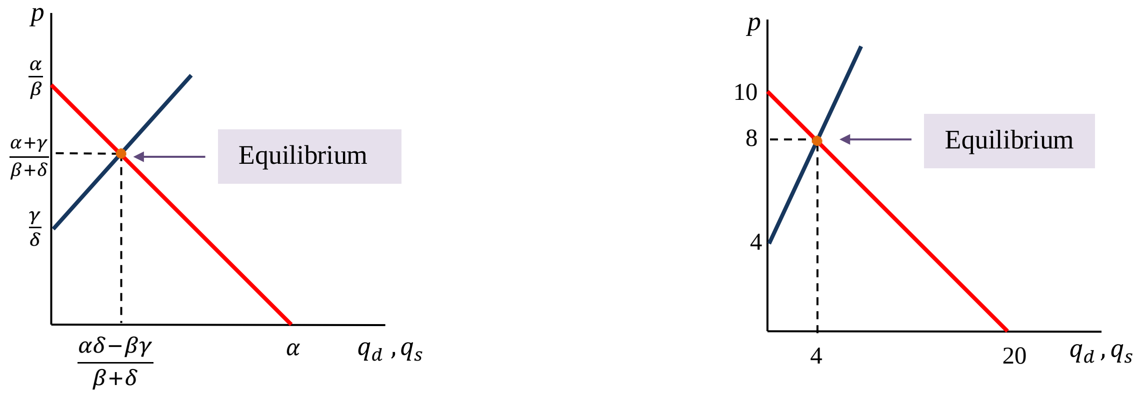

Figure 1

One question that might be posed when conducting a comparative static analysis of this model concerns what will happen in the market if households' incomes increase. Assuming that the good traded is a normal good an increase in households' incomes will mean that at each price households will demand more of the good. If at each price an additional quantity equal to τ (6 - an arbitrary increase) is demanded, the new demand function will take the following form. Note that this exogenous change has resulted in a change (increase) in the constant term of the equation that defines the demand function.

Problems

1. A market is described by the following model.

$$q_{dt} = 18 - 2p_t$$

$$q_{st} = -12 + 3p_{t-1}$$

where q_{dt} = quantity demanded in time period t

q_{st} = quantity supplied in time period t

p_t = price in time period t

On the assumption that in any time period price adjusts to clear the market, find the equation giving the time path of p_t if $p_0 = 5$. Comment on the stability of this market. Find the equation that gives the time path of quantity traded. Find the market price and quantity traded in time period 1 and time period 2, that is, (p_1, q_1) and (p_2, q_2). If $p_0 = 6$, what will the market price be when $t = 10$? Explain this result.

