(mathematics) √ inEconomics

WORKBOOK

Economic Applications 5

Contents

This workbook contains a set of problems for which solutions can be found using mathematical techniques that include calculus to optimise a function of two variables and total differentiation. The problems are written using functions that include polynomial (to degree two), rational, algebraic, exponential and logarithmic. Some functions are expressed in terms of parameters.

Economic applications include the total, average and marginal product of labour and capital, total, average and marginal revenue, profit maximisation for a multiproduct firm and for firms operating in an oligopoly, marginal utility, the own-price and cross-price elasticity of demand, substitute and complementary goods, the Keynesian national income model and the budget deficit.

There are 21 problems together with worked solutions. The answers to them are given separately.

Page border from an edition of 'The King of the Golden River' by John Ruskin published as a pamphlet by James Brodie Limited. Illustrator and publication date unknown.

Mathematical Symbols and Letters from the Greek Alphabet used in Workbooks				
Mathematical symbols		*Letters from the Greek Alphabet*		
Symbol	**Meaning**	**Capital**	**Lower case**	**Name**
$\therefore$	therefore	A	α	alpha
$\approx$	approximately equal to	B	β	beta
$\equiv$	identically equal to	Δ	δ	delta
$<$	less than	E	ε	epsilon
$>$	greater than	H	η	eta
$\leq$	less than or equal to	Θ	θ	theta
$\geq$	greater than or equal to	Λ	λ	lambda
$\Rightarrow$	implies	M	μ	mu
$\pm$	plus or minus	N	ν	nu
e	the exponential constant	Π	π	pi
∞	infinity	P	ρ	rho
		$\sum$	σ	sigma

Using this workbook

The purpose of this workbook is to provide a set of problems that involve different aspects of economics and that can be solved using the mathematical techniques described on page 1. The problems are arranged randomly in terms of the techniques needed to solve them to help develop an ability to determine the appropriate technique needed to solve a problem. However in general, problems towards the end of the workbook are more difficult to solve and often involve the application of more advanced mathematical techniques.

Worked solutions to the problems are given on pages 10-27. There is almost always more than one way of finding the solution to a problem. In most cases therefore, the worked solution given for a particular problem shows only one possible approach. There is no intention to suggest that the problem *has* to be solved using this approach or that it is necessarily a better approach than the alternatives. However, if there are two or more problems with the same structure (for example two problems that involve solving the same type of equation) the method shown in the worked solution for solving one of them may be different from the method shown for another. This is done to give additional information about solving the problem and to discourage repetition of one particular method of solution. Answers to all the problems are given on pages 28-31.

The best approach to using this workbook is to try solving some of the early problems and decide if they are appropriate to your level of mathematical ability. If they are too easy try some of the later problems. Solve a problem, check the solution, and then look at the answer given (pages 28-31). If your answer to the problem is not correct, and if *after some more work* you still cannot find the correct answer, look at the worked solution. There is nothing whatsoever to be gained by looking at the worked solution before a serious attempt has been made to try and solve the problem.

Since the problems in this workbook relate to different areas of economics some knowledge of the area of economics relevant to a particular problem is usually needed to solve it.

Problems

1. A firm faces an average revenue function of the following form where p and q represent price and quantity respectively.

$$p = \frac{360}{q+2} - 5$$

Find the total and marginal revenue functions for the firm. Draw the total revenue curve. For what level of output will total revenue be maximised? What is marginal revenue at this level of output?

2. The following data gives aggregate consumers' expenditure at different levels of aggregate income in an economy.

Aggregate income (£bn)	Consumers' expenditure (£bn)
0	98
100	170
200	242
300	314

(i) Find the consumption function for this economy on the assumption it is linear, and graph it. What is the value of the marginal propensity to consume?

(ii) If aggregate income is 600 what will consumers' expenditure be?

(iii) Find the savings function and graph it. What is the value of the marginal propensity to save? At what level of income will savings be zero?

3. Find the elasticity for each of the following supply functions. Comment on each elasticity.

$$q_s = f(p) = 8 + 2p$$

$$q_s = g(p) = 6p$$

$$q_s = h(p) = -4 + \tfrac{1}{3}p$$

where q_s = quantity supplied
p = price

4. In a market for two interrelated goods, X_1 and X_2, the demand functions are:

$$x_1 = f(p_1, p_2, y) = 48 - 3p_1 + 0.5p_2 + 0.8y$$

$$x_2 = f(p_1, p_2, y) = 20 - 2p_2 + 0.5p_1 + 0.45y$$

where x_i = quantity demanded of good X_i for $i = 1, 2$
p_i = price of good X_i for $i = 1, 2$
y = households' income

(i) Are X_1 and X_2 complementary or substitute goods?

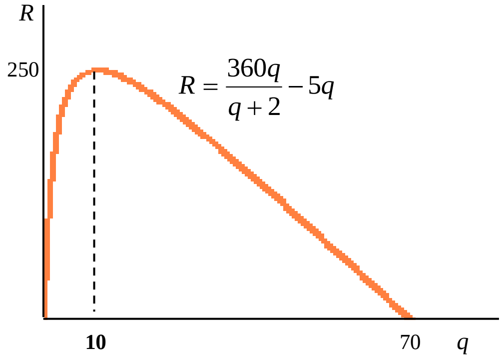
(ii) Are they normal or inferior goods?

Worked solutions

1. Let R represent total revenue.

$$R = pq = \left(\frac{360}{q+2} - 5\right)q = \frac{360q}{q+2} - 5q$$

$$\frac{dR}{dq} = \frac{(q+2)360 - 360q(1)}{(q+2)^2} - 5 = \frac{720}{(q+2)^2} - 5$$ Using the quotient rule



The level of output for which profit will be maximised can be found using calculus.

First-order condition for a stationary point:

$$\frac{dR}{dq} = \frac{720}{(q+2)^2} - 5 = 0$$

Solving this equation:

$$\frac{720}{(q+2)^2} = 5$$

$$(q+2)^2 = \frac{720}{5}$$

$$q = \pm(144)^{\frac{1}{2}} - 2$$

$$q = 10 \quad \text{or} \quad q = -14$$

Second-order condition:

$$\frac{dR}{dq} = (-2)\frac{720}{(q+2)^3} = \begin{cases} -\frac{5}{6} & \text{when } q = 10 \\ \frac{5}{6} & \text{when } q = -14 \end{cases}$$

Total revenue is maximised when $q = 10$. $\frac{dR}{dq} = \frac{720}{(q+2)^2} - 5 = \frac{720}{(10+2)^2} - 5 = 0$

2. (i) Let C represent consumers' expenditure and Y represent aggregate income. Since the consumption function is linear it must take the following form where a and b represent unknown parameters.

$$C = a + bY$$

Since all the pairs of values (Y, C) given in the table must satisfy this relationship taking any two pairs will give two equations in a and b from which the values of these parameters can be found. Taking the first two pairs:

$$98 = a + b0 \qquad (1)$$

$$170 = a + b100 \qquad (2)$$

This is a set of two linear simultaneous equations in the variables a and b. One way to solve this set of equations is using the substitution method as follows.

From (1): $a = 98$

Substituting for a in (2):

$$170 = 98 + b100$$

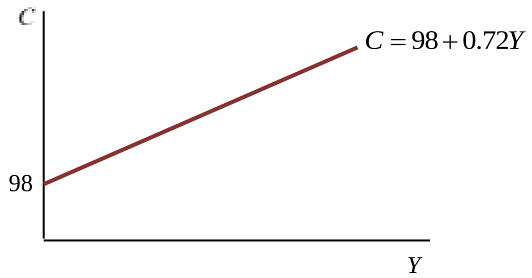
$$72 = b100$$

$$b = 0.72$$

Substituting for a and b in the consumption equation gives:

$$C = 98 + 0.72Y$$

The marginal propensity to consume is given by $\frac{dC}{dY} = 0.72$.



(ii) If $Y = 600$: $C = 98 + 0.72Y = 98 + 0.72(600) = 530$

(iii) Let S represent savings in this economy. On the assumption money is either spent on goods or saved it follows that $Y = C + S$. The savings function expresses saving as a function of income (Y). From the definitional equation $Y = C + S$:

$$S = Y - C$$

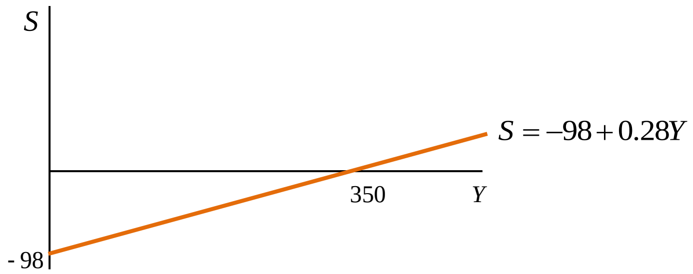
To express S in terms of Y only, substitute for C in terms of Y.

$$S = Y - C$$

$$= Y - (98 + 0.72Y)$$

$$= -98 + 0.28Y$$

The savings function is $S = -98 + 0.28Y$.



The marginal propensity to save is given by $\frac{dS}{dY} = 0.28$.

When $S = 0 : 0 = -98 + 0.28Y \quad \Rightarrow \quad Y = 350$

3. $\varepsilon_s = \frac{dq_s}{dp} \cdot \frac{p}{q_s}$

$q_s = f(p) = 8 + 2p$: $\quad \frac{dq_s}{dp} \cdot \frac{p}{q_s} = 2\left(\frac{p}{q_s}\right) = \frac{2p}{8+2p} = \frac{p}{4+p} \qquad \left(\text{or } \frac{q_s - 8}{q_s}\right)$

Since $p \geq 0$ and $p < 4 + p$, $0 \leq \varepsilon_s < 1$ so supply is inelastic at all prices.

$q_s = g(p) = 6p$: $\quad \frac{dq_s}{dp} \cdot \frac{p}{q_s} = 6\left(\frac{p}{q_s}\right) = \frac{6p}{6p} = 1$

Since $\varepsilon_s = 1$ supply has unit elasticity at all prices.

$q_s = h(p) = -4 + \frac{1}{3}p$: $\quad \frac{dq_s}{dp} \cdot \frac{p}{q_s} = \frac{1}{3}\left(\frac{p}{q_s}\right) = \frac{p}{-12+p} \qquad \left(\text{or } \frac{q_s + 4}{q_s}\right)$

To be economically meaningful $p \geq 0$ and $q_s \geq 0$. However ε_s is not defined for $q_s = 0$ so $q_s > 0 \quad \Rightarrow \quad p > 12$. For $p > 12$, $\varepsilon_s > 1$ so supply is elastic at all prices.

4. (i) $\frac{\partial x_1}{\partial p_2} = 0.5 \qquad \frac{\partial x_2}{\partial p_1} = 0.5$

Since these derivatives are positive these goods are substitutes.

(ii) $\frac{\partial x_1}{\partial y} = 0.8 \qquad \frac{\partial x_2}{\partial y} = 0.45$

These derivatives are both positive so both goods are normal goods.