(mathematics) inEconomics

WORKBOOK

Economic Applications 4

Contents

This workbook contains a set of problems that can be solved using the same set of mathematical techniques as used in the workbook on Economic Applications 3 but the problems in this workbook are generally harder. The mathematical techniques include solving linear and non-linear equations and sets of such equations and using calculus applied to a function of one variable. The rules of differentiation used include the product, quotient and chain rules, and the exponential function and logarithmic function rules. The functions used include polynomial (to degree three), algebraic, exponential and logarithmic. Some functions are expressed in terms of parameters and some are expressed generally.

The applications include the perfectly competitive market model, the elasticity of demand, consumer and producer surplus, the Keynesian model, the budget deficit, a balanced budget, total, average and marginal product of labour, the marginal revenue product of labour, the supply of labour, profit and revenue maximisation, cost minimisation and the discrete compound growth model.

There are 20 problems together with worked solutions. Answers to these problems are given separately.

Page border from an edition of 'The King of the Golden River' by John Ruskin published as a pamphlet by James Brodie Limited. Illustrator and publication date unknown.

Copyright © 2017 by Susan Evans

Mathematical Symbols and Letters from the Greek Alphabet used in Workbooks				
Mathematical symbols		***Letters from the Greek Alphabet***		
Symbol	**Meaning**	**Capital**	**Lower case**	**Name**
$\therefore$	therefore	A	α	alpha
$\approx$	approximately equal to	B	β	beta
$\equiv$	identically equal to	Δ	δ	delta
$<$	less than	E	ε	epsilon
$>$	greater than	H	η	eta
$\leq$	less than or equal to	Θ	θ	theta
$\geq$	greater than or equal to	Λ	λ	lambda
$\Rightarrow$	implies	M	μ	mu
$\pm$	plus or minus	N	ν	nu
e	the exponential constant	Π	π	pi
∞	infinity	P	ρ	rho
		$\sum$	σ	sigma

Using this workbook

The purpose of this workbook is to provide a set of problems that involve different aspects of economics and that can be solved using the mathematical techniques described on page 1. The problems are arranged randomly in terms of the techniques needed to solve them to help develop an ability to determine the appropriate technique needed to solve a problem. In general, problems towards the end of the workbook are more difficult to solve.

Worked solutions to the problems are given on pages 10-28. There is almost always more than one way of finding the solution to a problem. In most cases therefore, the worked solution given for a particular problem shows only one possible approach. There is no intention to suggest that the problem *has* to be solved using this approach or that it is necessarily a better approach than the alternatives. However, if there are two or more problems with the same structure (for example two problems that involve solving the same type of equation) the method shown in the worked solution for solving one of them may be different from the method shown for another. This is done to give additional information about solving the problem and to discourage repetition of one particular method of solution. Answers to all the problems are given on pages 29-33.

The best approach to using this workbook is to try solving some of the early problems and decide if they are appropriate to your level of mathematical ability. If they are too easy try some of the later problems. Solve a problem, check the solution, and then look at the answer given (pages 29-33.). If your answer to the problem is not correct, and if *after some more work* you still cannot find the correct answer, look at the worked solution. There is nothing whatsoever to be gained by looking at the worked solution before a serious attempt has been made to try and solve the problem.

Since the problems in this workbook relate to different areas of economics some knowledge of the area of economics relevant to a particular problem is usually needed to solve it.

Copyright © 2017 by Susan Evans

Problems

1. A firm has a short-run average total cost (A_C) function of the form:

$$A_C = \frac{2}{5}q + \frac{90}{q} + 18$$

where q = output

(i) Find the level of output at which short-run average total cost is minimised.

(ii)Show that marginal cost is equal to average total cost at the level of output in (i).

2. Aggregate expenditure by consumers in an economy is given by:

$$C = 88 + 0.85Y_d$$

where Y_d = disposable income
C = consumers' expenditure

Investment expenditure by firms is 200.

(i) If the government is raising taxes on income (Y) at a rate of 20% find consumers' expenditure as a function of Y and thus find $\frac{dC}{dY}$. What does $\frac{dC}{dY}$ represent?

(ii) If the government raise taxes at a rate of 20% and spends 80, what is the equilibrium level of income in the economy? How much will consumers' spend at the equilibrium level of income?

(iii) Is there a budget deficit or surplus at the level of income in (ii)?

(iv) With the tax rate remaining at 20%, if the government decides to balance the budget by changing expenditure, how much will it have to spend and what will the impact be on income in the economy? Represent government expenditure and revenue graphically showing it before and after the change in government expenditure.

(v) With expenditure of 80 if the government decides to balance the budget by changing the tax rate, what tax rate will it need to levy and what will the impact be on income in the economy? Represent government expenditure and revenue graphically showing it before and after the change in government expenditure.

3. A short-run production function takes the form:

$$q = q(L) = -0.5L^3 + 18L^2 + 168L$$

where q = output (units)
L = labour input

(i) Find the values of L for which the marginal product of labour is increasing, constant and decreasing. What is the value of the marginal product of labour when it is constant?

(ii) Draw the average and marginal product of labour curves on the same diagram. For what positive value of L is average product equal to marginal product and what value do they take at this value of L? What significant feature occurs at the point where average and marginal product are equal?

Copyright © 2017 by Susan Evans

Worked solutions

1. (i) $$A_C = \frac{2}{5}q + \frac{90}{q} + 18$$

First-order condition for a stationary point:

$$\frac{dA_C}{dq} = \frac{2}{5} - \frac{90}{q^2} = 0 \qquad \Rightarrow \qquad q^2 = 225 \qquad \Rightarrow \qquad q = \pm 15$$

Second-order condition:

$$\frac{d^2A_C}{dq^2} = \frac{180}{q^3} = \begin{cases} \frac{4}{75} > 0 & \text{when} \quad q = 15 \\ -\frac{4}{75} < 0 & \text{when} \quad q = -15 \end{cases}$$

Short-run average total cost is minimised when $q = 15$.

(ii) Marginal cost is $\frac{dC}{dq}$ where C is total cost.

$$C = qA_C = q\left(\frac{2}{5}q + \frac{90}{q} + 18\right) = \frac{2}{5}q^2 + 90 + 18q$$

$$\frac{dC}{dq} = \frac{4}{5}q + 18$$

When $q = 15$:

$$\frac{dC}{dq} = \frac{4}{5}q + 18 = \frac{4}{5}(15) + 18 = 30$$

$$A_C = \frac{2}{5}q + \frac{90}{q} + 18 = \frac{2}{5}(15) + \frac{90}{15} + 18 = 30$$

2. (i) $C = 88 + 0.85Y_d$ $\qquad$ $Y_d = Y - tY$ where t is the proportionate tax rate

Since the tax rate is 20%: $t = 0.2$

Substituting $t = 0.2$ in the equation that defines Y_d: $Y_d = Y - tY = Y - 0.2Y = 0.8Y$

Substituting in the consumption function for Y_d:

$$C = 88 + 0.85Y_d = 88 + 0.85(0.8Y) = 88 + 0.68Y$$

$\frac{dC}{dY} = 0.68$ $\qquad$ This derivative is the marginal propensity to consume.

(ii) The model of this economy can be written in terms of the following equations.

$C = 88 + 0.68Y$ (1)

$I = 200$ (2)

$G = 80$ (3)

$Y = C + I + G$ (4)

Copyright © 2017 by Susan Evans

This set of equations can be reduced to one equation in Y by substituting for C, I and G from (1), (2) and (3) , in (4):

$$Y = 88 + 0.68Y + 200 + 80$$

$$Y - 0.68Y = 368$$

$$Y(1 - 0.68) = 368$$

$$Y(0.32) = 368$$

$$Y = \frac{368}{0.32}$$

$$Y = 1{,}150$$

Let Y_e and C_e represent the equilibrium level of income and consumers' expenditure, respectively.

When $Y_e = 1{,}150$: $C_e = 88 + 0.68Y_e = 88 + 0.68(1{,}150) = 870$

(iii) Let T represent tax revenue to the government.

If $G - T < 0$ there is a budget surplus.

If $G - T > 0$ there is a budget deficit.

$G = 80$, $T = 0.2Y = 0.2(1{,}150) = 230$

$G - T = 80 - 230 = -150 < 0$ At equilibrium there is a budget surplus of 150.

(iv) For the budget to balance $G - T = 0 \quad \Rightarrow \quad G = T \quad \Rightarrow \quad G = 0.2Y$

Substituting $G = 0.2Y$ in place of 80 in the equilibrium equation:

$$Y = 88 + 0.68Y + 200 + 0.2Y$$

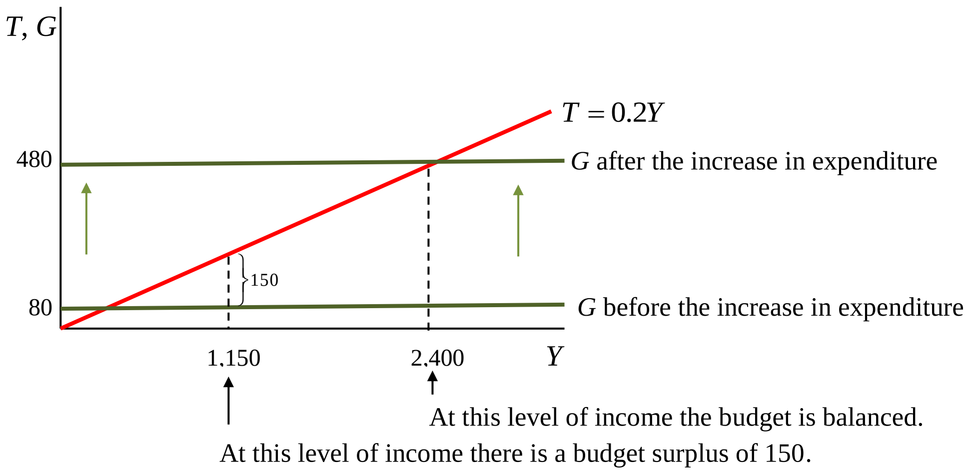
$$Y - 0.68Y - 0.2Y = 288$$

$$Y(1 - 0.68 - 0.2) = 288$$

$$Y = 2{,}400$$

When $Y = 2{,}400$: $G = 0.2Y = 0.2(2{,}400) = 480$

The equilibrium level of income in the economy has increased by $2{,}400 - 1{,}150 = 1{,}250$.



Copyright © 2017 by Susan Evans

(v) Since G is to remain at 80 $G = T \quad \Rightarrow \quad 80 = tY$

Substituting 80 for tY in the equation that defines Y_d: $Y_d = Y - tY = Y - 80$

Substituting in the consumption function for Y_d:

$$C = 48 + 0.85Y_d = 88 + 0.85(Y - 80) = 20 + 0.85Y$$

This model can be written:

$$C = 20 + 0.85Y \qquad (1)$$

$$I = 200 \qquad (2)$$

$$G = 80 \qquad (3)$$

$$Y = C + I + G \qquad (4)$$

$$Y = 20 + 0.85Y + 200 + 80$$

$$Y - 0.85Y = 300$$

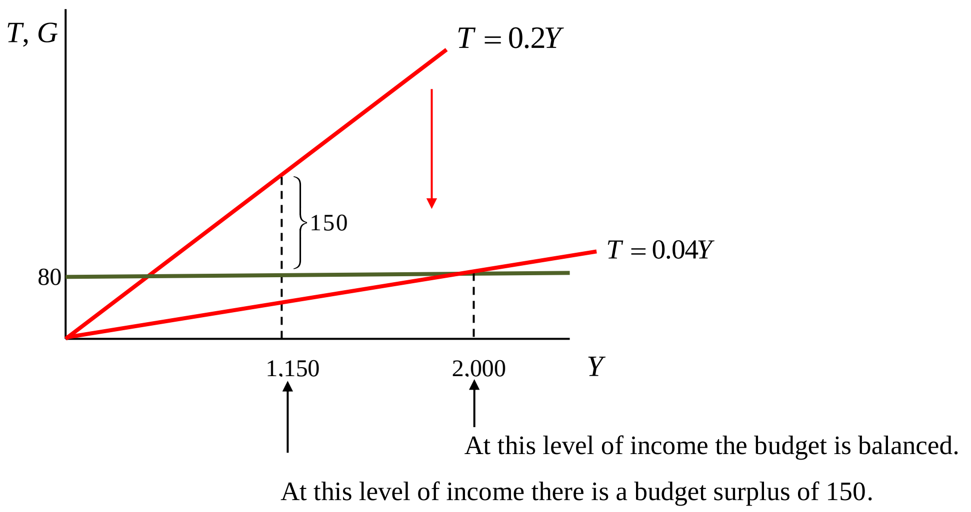
$$Y = 2{,}000$$

When $Y = 2{,}000$ the proportionate tax rate must satisfy the condition that $80 = tY$

$$80 = tY \quad \Rightarrow \quad 80 = t(2{,}000) \quad \Rightarrow \quad t = \frac{80}{2{,}000} = 0.04$$

To balance the budget the government should reduce the tax rate to 4%.

The equilibrium level of income in the economy has increased by $2{,}000 - 1{,}250 = 850$.



Copyright © 2017 by Susan Evans