(mathematics) √inEconomics

WORKBOOK

Economic Applications 1

Contents

This workbook contains a set of problems that can be solved using some basic mathematical techniques that include solving linear and non-linear equations and sets of linear simultaneous equations containing two variables. No calculus is used. The functions used include piecewise linear, polynomial (constant, linear, quadratic), hyperbolic, algebraic, exponential and logarithmic.

Economic applications include demand and supply, perfect competition, consumer surplus, lump-sum and proportionate taxation, gross and net income, government revenue, utility functions and indifference curves, total, average and marginal revenue, total and average cost, profit maximisation, revenue maximisation, the break-even point, the Keynesian model, the average and marginal propensity to consume, the arithmetic, geometric and harmonic mean, net present value, and discrete and continuous compound growth models.

There are 30 problems together with worked solutions. Answers to these problems are given separately.

Page border from an edition of 'The King of the Golden River' by John Ruskin published as a pamphlet by James Brodie Limited. Illustrator and publication date unknown.

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Mathematical Symbols and Letters from the Greek Alphabet used in Workbooks				
Mathematical symbols		***Letters from the Greek Alphabet***		
Symbol	**Meaning**	**Capital**	**Lower case**	**Name**
$\therefore$	therefore	A	α	alpha
$\approx$	approximately equal to	B	β	beta
$\equiv$	identically equal to	Δ	δ	delta
$<$	less than	E	ε	epsilon
$>$	greater than	H	η	eta
$\leq$	less than or equal to	Θ	θ	theta
$\geq$	greater than or equal to	Λ	λ	lambda
$\Rightarrow$	implies	M	μ	mu
$\pm$	plus or minus	N	ν	nu
e	the exponential constant	Π	π	pi
∞	infinity	P	ρ	rho
		$\sum$	σ	sigma

Using this workbook

The purpose of this workbook is to provide a set of problems that involve different aspects of economics and that can be solved using some basic mathematical techniques (see page 1). The problems are arranged randomly in terms of the techniques needed to solve them to help develop an ability to determine the appropriate technique needed to solve a problem. In general, problems towards the end of the workbook are more difficult to solve.

Worked solutions to the problems are given on pages 10-24. There is almost always more than one way of finding the solution to a problem. In most cases therefore, the worked solution given for a particular problem shows only one possible approach. There is no intention to suggest that the problem *has* to be solved using this approach or that it is necessarily a better approach than the alternatives. If there are two or more problems with the same structure (for example two problems that involve solving the same type of equation) the worked solution given may show a different approach to solving each problem so as to highlight the fact that there are different methods by which a solution can be found. Answers to all the problems are given on pages 25-28.

The best approach to using this workbook is to try solving some of the early problems and decide if they are appropriate to your level of mathematical ability. If they are too easy try some of the later problems, alternatively try the problems in the workbook on Economic Applications 2 which includes some harder problems whose solutions can be found using the same set of mathematical techniques as used in this workbook. Solve a problem, check the solution, and then look at the answer given (pages 25-28). If your answer to the problem is not correct, and if *after some more work* you still cannot find the correct answer, look at the worked solution. There is nothing whatsoever to be gained by looking at the worked solution before a serious attempt has been made to try and solve the problem.

Since the problems in this workbook relate to different areas of economics some knowledge of the area of economics relevant to a particular problem is usually needed to solve it.

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Problems

1. A worker receives take-home pay of £348.58 per week. If income tax, national insurance and a payment into a pension fund account for 27.5% of gross income, what is this worker's gross income?

2. A firm has agreed to pay a new employee an income of £*Y* per annum for the first year of employment. If the employee remains in the job after a year, in each subsequent year an annual increment of £1,200 will be paid. If the employee remains in the employment of the firm for three years, the total income received in that time will be £75,600. What is the value of *Y*?

3. Let *x* represent average household income (£) in an economy and *y* represent the income of an individual household. Define *y* for each of the following households.

   (i) A household whose income is £2,500 above average income.

   (ii) A household whose income is 15% above average income.

   (iii) A household whose income is 60% of average income.

   (iv) A household whose income is 25% below average income.

   (v) A household whose income is £4,000 less than twice average income.

4. The following information relating to the UK describes some tax policy measures announced in the Budget in March 2016 and subsequently published by HM Revenue and Customs.

1.	**Personal tax and benefits**	
1.1		**Income Tax bands of taxable income (£ per year)**
	Tax rate	**Tax year 2016 - 2017**
	Basic rate	£0 - 32,000
	Higher rate	£32,001-150,000
	Additional rate	Over £150,000
1.2		**Income Tax Rates**
	Basic rate	20%
	Higher rate	40%
	Additional rate	45%
1.6		**Income Tax allowances 2016 - 17 (£ per year)**
	Personal allowance £11,000	

Let *x* represent the earned income of an employee in the UK to which the rates and allowance given in the table above apply. Define the tax function $T = t(x)$ where T represents the tax paid on earned income in the tax year 2016-2017. How much tax will an employee earning £64,000 pay?

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Worked solutions

1. Let G represent gross income. Since deductions from gross income account for 27.5 % of it, the worker's take-home pay (net income) constitutes $(100-27.5)\% = 72.5\%$ of gross income so the following equation holds.

$$0.725G = 348.58$$

$$G = \frac{348.58}{0.725}$$

$$G = 480.8$$

The worker's gross income is £480.8.

2.

Year	Income
1	Y
2	$Y + 1,200$
3	$Y + 1,200 + 1,200 = Y + 2(1,200)$
	$\sum 75,600$

$$Y + Y + 1,200 + Y + 2(1,200) = 75,600$$

$$3Y = 75,600 - 3,600$$

$$Y = 24,000$$

3. (i) $y = x + 2,500$

(ii) $y = x + (\frac{15}{100})x = x(1+0.15) = 1.15x$

(iii) $y = (\frac{60}{100})x = 0.6x$

(iv) $y = x - (\frac{25}{100})x = x(1-0.25) = 0.75x$

(v) $y = 2x - 4,000$

4. The employee has a personal allowance of up to £11,000 so on earned income up to £11,000 the employee will pay no tax. Treating x as a continuous variable:

$$T = t(x) = 0 \qquad x \le 11,000$$

On income above £11,000 up to $11,000 + 32,000 = 43,000$ the employee will pay 20% in tax:

$$T = t(x) = 0.2(x - 11,000) = 0.2x - 2,200 \qquad 11,000 < x \le 43,000$$

On income from $£43,001 - £161,000$ ($£150,000 + £11,000$) the employee will pay 40% in tax:

$$T = t(x) = 0.2(43,000 - 11,000) + 0.4(x - 43,000)$$

$$= 0.4x - 10,800 \qquad 43,000 < x \le 161,000$$

On income above £161,000 ($£150,000 + £11,000$) the employee will pay 45% in tax:

$$T = t(x) = 0.2(43,000 - 11,000) + 0.4(161,000 - 43,000) + 0.45(x - 161,000)$$

$$= 0.45x - 18,850 \qquad x > 161,000$$

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$T = t(x)$	
$T = 0$	$x \le 11{,}000$
$T = 0.2x - 2{,}200$	$11{,}000 < x \le 43{,}000$
$T = 0.4x - 10{,}800$	$43{,}000 < x \le 161{,}000$
$T = 0.45x - 18{,}850$	$x > 161{,}000$

If an employee earns £64,000: $T = 0.4x - 10{,}800 = 0.4(64{,}000) - 10{,}800 = 14{,}800$

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