Mathematical Methods in Economics: Problems and Solutions

Chapter 9

Problems on Integration

9.1 A firm has a marginal cost function of the form:

$$C_m = 5 + 8q + 32q^2$$

where $C_m =$ marginal cost

$q =$ output

If the firm's fixed costs are 180, find the total cost function.

9.2 The inverse demand function for a good is given by:

$$p = f(q) = 70 - 9q - q^2$$

where $p =$ price (£)

$q =$ quantity demanded (units)

Find consumer surplus if market price is £34. How does consumer surplus change if the market price falls to £18?

9.3 A perfectly competitive industry is described by the following inverse demand and supply functions:

$$p = f(q_d) = 200 - 5q_d$$

$$p = g(q_s) = 32 + 2q_s^2$$

where $p =$ price (£)

$q_d =$ quantity demanded (units)

$q_s =$ quantity supplied (units)

(i) Determine the quantity that will be traded and the price at which the product will be sold if the market is in equilibrium.

(ii) Suppose that an individual firm buys up all the firms in the industry so that the industry becomes a monopoly. Costs do not change as a result of this take-over. The objective of the firm is to maximise profit. Determine the output that the monopolist will produce and the price at which it will be sold. What is the change in consumer surplus resulting from the change in industrial structure?

9.4 A firm has a marginal revenue function:

$$r(q) = 300 - 5q^{\frac{2}{3}}$$

where $q =$ output (units)

If the firm is breaking even and producing 729 units of output, what are its total costs?

9.5 A market demand function is given by:

$$q_d = f(q_d) = \frac{104{,}976}{p^4}$$

where p = price (£)

q_d = quantity demanded (units)

(i) Find consumer surplus if the market price of the product is £9.

(ii) Find consumer surplus if the market price of the product is £6.

(iii) Find consumer surplus as a proportion of total expenditure in (i) and in (ii).

9.6 A profit-maximising monopolist has a marginal revenue function given by:

$$r(q) = 360 - \tfrac{2}{3}q$$

where $r(q)$ = marginal revenue (£)

q = output (units)

The monopolist is receiving total revenue of £63,492.

(i) What level of output is the monopolist producing?

(ii) What is consumer surplus?

(iii) Find the elasticity of demand at the current level of output.

9.7 A firm operating in a perfectly competitive market is producing the profit-maximising level of output and making normal profits. The price in the market is 76. The marginal cost function for the firm is given by:

$$\frac{dC}{dq} = 0.375q^2 + 22$$

where C = total cost

q = output

(i) What output is the firm producing?

(ii) What are the firm's total variable and total fixed costs?

(iii) What is the equation of the average total cost function?

(iv) Represent the average total, average fixed and marginal cost functions graphically.

9.8 A firm operating in a perfectly competitive market is in equilibrium producing 8 units of output and making normal profits. The marginal cost function of the firm is given by:

$$m(q) = 1.35q^2 + b$$

where q = output

b is a constant

(i) Total variable costs for the firm at the current level of output are 646.4, find the value of b.

(ii) What is the market price and what are the firm's fixed costs?