(mathematics) inEconomics

Mathematical Methods in Economics: Problems and Solutions

Chapter 8

Problems on Constrained Optimisation II

8.1 A consumer has a utility function:

$$u(x, y) = x^{\alpha} y^{\beta} \qquad 0 < \alpha, \beta < 1, \;\; \alpha + \beta = 1$$

where x = quantity of good X (units)
y = quantity of good Y (units)

The consumer has an income of £m. The price of X is £p_x and that of Y is £p_y. Show that the consumer will maximise utility by consuming where:

$$x = \frac{\alpha m}{p_x}$$

$$y = \frac{\beta m}{p_y}$$

8.2 A monopolist is selling output to two different countries with inverse demand functions given by:

$$p_1 = f(q_1) = 20 - \tfrac{1}{4} q_1$$

$$p_2 = g(q_2) = 17 - \tfrac{1}{2} q_2$$

where p_i = price (£) in country i, for $i = 1, 2$
q_i = quantity demanded in country i (units)

The total cost function for the monopolist is given by:

$$C = c(q) = 115 + 2q$$

where C = total cost (£)
q = output (units)

(i) Determine the output the monopolist should sell in each market to maximise profit and the level of profit that the monopolist will make.

(ii) Suppose that as a consequence of import restrictions it is necessary for the monopolist to set the price in country 1 at £3 more than the price in country 2. How much will the monopolist sell in each country to maximise profit and satisfy this constraint? How does profit differ from that in (i)?

8.3 A consumer obtains utility from two goods X and Y and has a utility function of the form:

$$u = f(x, y) = x + \ln y$$

where u = index of utility
x = quantity of good X (units)
y = quantity of good Y (units)

The consumer has an income of £m and the prices at which X and Y can be bought are £p_x and £p_y respectively.

(i) Find the demand function for X and for Y.

(ii) Find the degree of homogeneity of the demand function for good X and demonstrate that Euler's theorem holds.

(iii) Does this consumer suffer from money illusion?

(iv) Find the own-price elasticity of demand and the income elasticity for good X.

8.4 A consumer has the following utility function:

$$U = u(x_1, x_2) = x_1 x_2^2$$

where U = level of utility
x_1 = quantity of good 1(units)
x_2 = quantity of good 2 (units)

The price of good 1 is $£p_1$ and that of good 2 is $£p_2$. The consumer's income is £*I*.

(i) Show that the consumer maximises utility where:

$$\frac{\text{marginal utility of good 1}}{p_1} = \frac{\text{marginal utility of good 2}}{p_2}$$

(ii) Derive the demand function for goods 1 and for good 2.

(iii) Find the elasticities of demand for good 2.

(iv) Draw the demand curve for good 2 when the consumer has an income of £90.

(v) If the utility function had been $U = v(x_1, x_2) = \ln x_1 + 2\ln x_2$, how would the demand functions differ from those in (ii)?

8.5 The preferences of a consumer can be expressed by the utility function:

$$u = f(x, y) = \log_e x^\alpha y^\beta$$

where u = index of utility
x = quantity of good X (units)
y = quantity of good Y (units)

(i) Find the marginal utilities of the two goods. What constraints must be imposed on the values of the parameters in this utility function in order for the marginal utilities to be diminishing?

(ii) If the price of good X is $£p_x$, the price of good Y is $£p_y$ and the consumer has an income of £*I*, find the demand functions for goods X and Y.

(iii) On what does the value of the marginal utility of income depend here? How will the marginal utility of money at B compare with its value at A in the following diagram?