(mathematics) $\sqrt{\ inEconomics}$

Mathematical Methods in Economics: Problems and Solutions

Chapter 2

Problems on Equations and Functions II

2.1 A perfectly competitive market is described by the following functions:

$$q_d = f(p) = \frac{8}{p^{\frac{1}{2}}}$$

$$q_s = g(p) = p$$

where q_d = quantity demanded (units)

q_s = quantity supplied (units)

p = price (£)

(i) Determine the equilibrium price and quantity traded in this market.

(ii) Represent this market graphically.

(iii) In order to increase consumption of this good, the government decides to subsidise consumers by paying a lump-sum subsidy per unit of the good. This policy results in a new equilibrium quantity of 10. What subsidy does the government pay to consumers?

2.2 A firm's total variable costs are given by the following equation where C represents total variable cost and q represents output.

$$C = f(q) = aq^{\alpha}$$

The firm has observed the following costs associated with the given levels of output:

Output q	Total variable costs C
2	20
4	80

(i) Find the equation of the total variable cost function.

(ii) If the firms' fixed costs are 40, find the average fixed, average variable, and average total cost function. To what class do these functions all belong?

2.3 The demand and supply functions for a particular commodity are given by:

$$q_d = (19 - p)^{\frac{1}{2}}$$

$$q_s = -\tfrac{7}{2} + \tfrac{5}{2}p$$

where q_d = quantity demanded

q_s = quantity supplied

p = price

(i) Find the equilibrium price and quantity.

(ii) Suppose consumers' tastes change and the new demand function has a constant elasticity of unity. If market equilibrium remains the same as in (i), find the new demand equation.

(iii) If after tastes change the government decides to impose a proportionate tax at a rate of t on suppliers, find the equilibrium price as a function of t.

2.4 A producer has fixed quantities of capital and labour and is using these inputs to manufacture two products, product 1 and product 2. The product transformation curve for this producer is given by:

$$x_1 = \left(8{,}000 - \frac{x_2}{4}\right)^{\frac{1}{3}}$$

where x_1 = output of product 1(units)
x_2 = output of product 2 (units)

(i) If the producer uses all inputs to produce product 1, what is the efficient level of production?

(ii) The producer has an order for 35,000 units of product 2; will it be possible to meet this order?

(iii) Give three different levels of output that the producer could produce if production is efficient.

(iv) Following technological developments, if resources are used solely in the production of product 1 output will be 20% above the previous maximum level of output. Alternatively, if the producer decides to produce only product 2 output will be 8% above the previous maximum. If the new product transformation curve takes the form:

$$x_1 = (a - bx_2)^{\frac{1}{3}}$$

Find the values of *a* and *b*.

2.5 Consider the model:

$$q_d = \alpha + \beta p$$

$$q_s = \gamma + \delta p$$

where q_d = quantity demanded
q_s = quantity supplied
p = price
$\alpha, \beta, \gamma, \delta$ are constants

The government imposes a tax of *T* per unit on suppliers.

(i) Find the equilibrium price and quantity as functions of *T*.

(ii) How do the equilibrium price and quantity change as the tax changes? Determine the likely direction of change in each case.

