(mathematics) inEconomics

Mathematical Methods in Economics: Problems and Solutions

Chapter 18

Worked Solutions to Problems on Integration

9.1 Let C = total cost.[1]

$$C=\int C_m dq=\int(5+8q+33q^2)dq=5q+\tfrac{8}{2}q^2+\tfrac{33}{3}q^3+C_0$$ Using the power rule

However, fixed costs = 180 so when q = 0, C = 180:

$$180=5(0)+4(0)^2+11(0)^3+C_0 \quad\Rightarrow\quad C_0=180$$

Therefore the total cost function is given by:

$$C=5q+4q^2+11q^3+180$$

9.2 Consumer surplus = $\int_0^{q_0} f(q)dq-34q_0=\int_0^{q_0}(70-9q-q^2)dq-34q_0$

where $q_0=$ quantity demanded at a price of £34

To find q_0 when p = 34: $34=70-9q-q^2 \quad\Rightarrow\quad q=3$ or $q=-12$

To be economically meaningful q = 3.

Consumer surplus = $\int_0^{3}(70-9q-q^2)dq-34(3)=\left[70q-\frac{9q^2}{2}-\frac{q^3}{3}\right]_0^3-102$

$$=\left[70(3)-\frac{9(3)^2}{2}-\frac{(3)^3}{3}\right]-[0]-102=58.5$$

9.3 (i)

$$p=200-5q_d \qquad (1)$$

$$p=32+2q_s^2 \qquad (2)$$

For equilibrium: $q_d=q_s$

One way to solve this set of three linear equations in three variables, q_d, q_s and p, is to substitute q_s for q_d from the equilibrium condition in (1):

$$p=200-5q_s \qquad (1a)$$

Substitute for p from (1a) in (2) and rearrange:

$$168-5q_s-2q_s^2=0$$

Solving this quadratic equation gives $q_s=8$ as the economically meaningful solution. From the equilibrium condition $q_d=8$. When $q_d=8$, from the demand function, $p=200-5(8)=160$. When the market is in equilibrium 8 units of output will be sold at a price of 160.

(ii) Let R = total revenue and C = total cost. $\pi=R-C$

$$R=pq=(200-5q)q=200q-5q^2$$

Since a perfectly competitive market became a monopoly, the marginal cost curve of the monopolist is given by:[2]

$$\frac{dC}{dq} = 32 + 2q^2 \qquad \Rightarrow \qquad C = \int (32 + 2q^2)\,dq = 32q + \frac{2q^3}{3} + C_0$$

The profit function will be given by:

$$\pi = 200q - 5q^2 - \left(32q + \frac{2q^3}{3} + C_0\right) = 168q - 5q^2 - \frac{2q^3}{3} - C_0$$

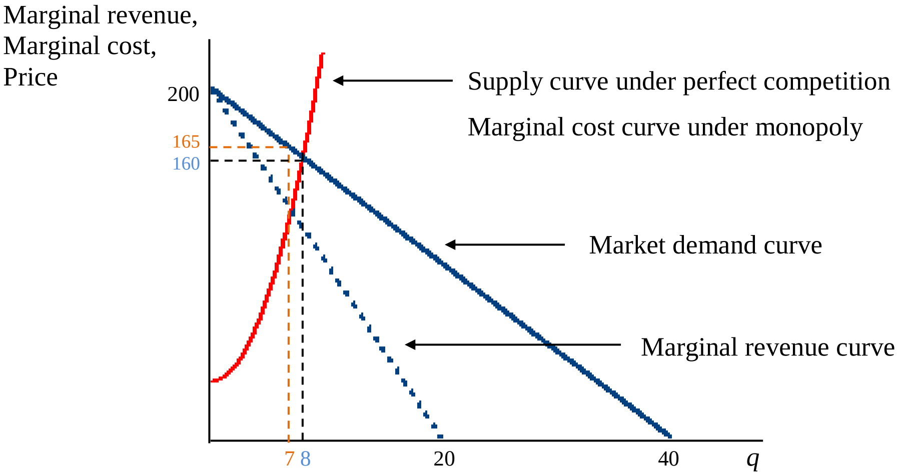
First-order condition for a stationary point:

$$\frac{d\pi}{dq} = 168 - 10q - 2q^2 = 0 \qquad \Rightarrow \qquad q = 7 \;\text{ or }\; q = -12$$

Second-order condition:

$$\frac{d^2\pi}{dq^2} = -10 - 4q = -38 < 0 \quad \text{when } q = 7$$

Profit is maximised when $q = 7$ and $p = 200 - 5(7) = 165$.[3]



Under perfect competition:

Consumer surplus $= \frac{1}{2} \times 8 \times (200 - 160) = 160$

Under monopoly:

Consumer surplus $= \frac{1}{2} \times 7 \times (200 - 165) = 122.5$

Change in consumer surplus $= 122.5 - 160 = -37.5$ [4]

