Mathematical Methods in Economics: Problems and Solutions

Chapter 13

Worked Solutions to Problems on Optimising a Function of One Variable

4.1 (i) Let π represent profit, R total revenue and C, total cost. $\pi = R - C$.

$$C = c(q) = 82.5 + 2q + 0.1q^2$$

The most direct way of determining the level of output that maximises profit is to express π as a function of q. The cost function is already expressed in terms of q so express revenue also in terms of q.

$$R = r(q) = \text{price} \times \text{quantity} = pq = (20 - 0.2q)q = 20q - 0.2q^2$$

$$\pi = R - C = 20q - 0.2q^2 - (82.5 + 2q + 0.1q^2) = 18q - 0.3q^2 - 82.5$$

Calculus can be used to determine the value of q that maximises this function.[1]

First-order condition for a stationary point:

$$\frac{d\pi}{dq} = 18 - 0.3(2)q = 0 \quad \Rightarrow \quad 18 = 0.6q \quad \Rightarrow \quad q = \frac{18}{0.6} = 30$$

Second-order condition:

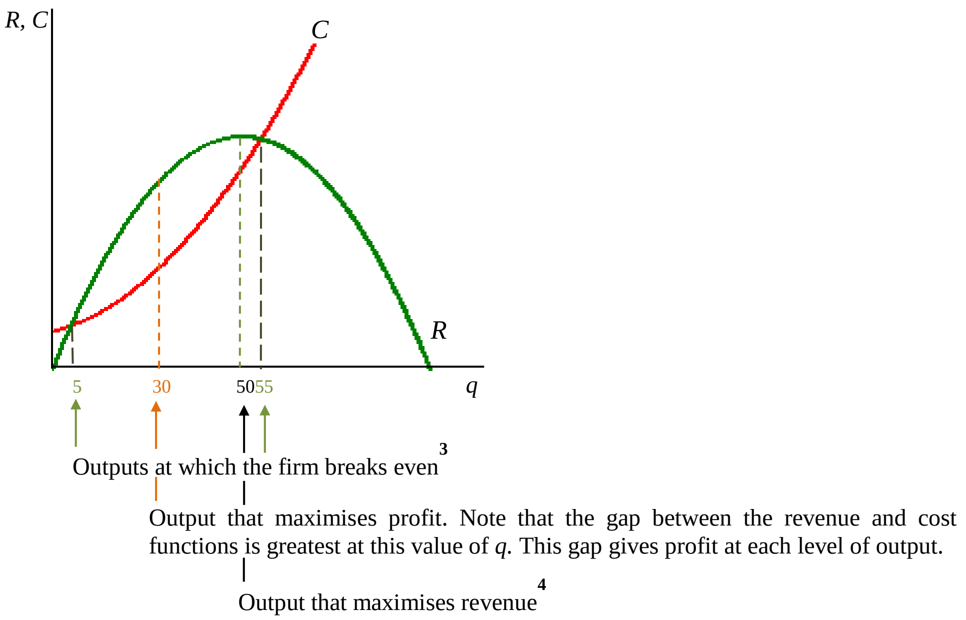
$$\frac{d^2\pi}{dq^2} = -0.6 < 0 \quad \text{profit is maximised at } q = 30$$

When q = 30: $p = 20 - 0.2(30) = 14$

(ii)[2] Marginal cost: $\frac{dC}{dq} = 2 + 0.1(2)q = 2 + 0.2(30) = 8$ when $q = 30$

Marginal revenue: $\frac{dR}{dq} = 20 - 0.2(2)q = 20 - 0.4(30) = 8$ when $q = 30$

(iii)



4.2 Let R = total revenue. $\pi = R - C$.

$$R = pq = (520 - q)q = 520q - q^2$$

$$\pi = (520q - q^2) - (1{,}470 + 24q + q^2) = -1{,}470 + 496q - 2q^2$$

First-order condition for a stationary point:

$$\frac{d\pi}{dq} = 496 - 4q = 0 \quad \Rightarrow \quad q = 124$$

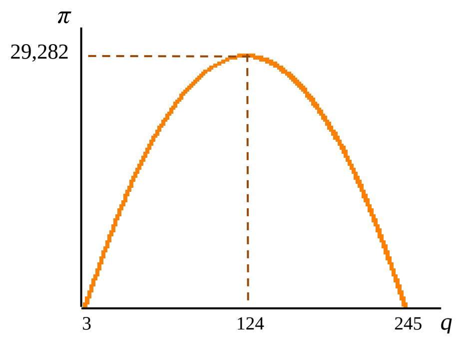
Second-order condition:

$$\frac{d^2\pi}{dq^2} = -4 < 0 \qquad \text{profit is maximised when } q = 124$$

Marginal revenue: $\dfrac{dR}{dq} = 520 - 2q = 272$ at $q = 124$

Marginal cost: $\dfrac{dC}{dq} = 24 + 2q = 272$ at $q = 124$

Profit function



The total cost function and total revenue function