(mathematics) √inEconomics

Mathematical Methods in Economics: Problems and Solutions

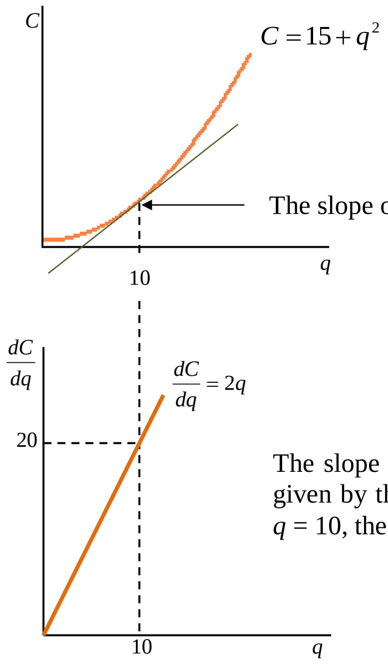
Chapter 12

Worked Solutions to Problems on Differentiating a Function of One Variable

3.1 Marginal cost[1] is given by $\frac{dC}{dq} = 2q$.

Using the power rule

The total and marginal cost curves are shown in the diagram below



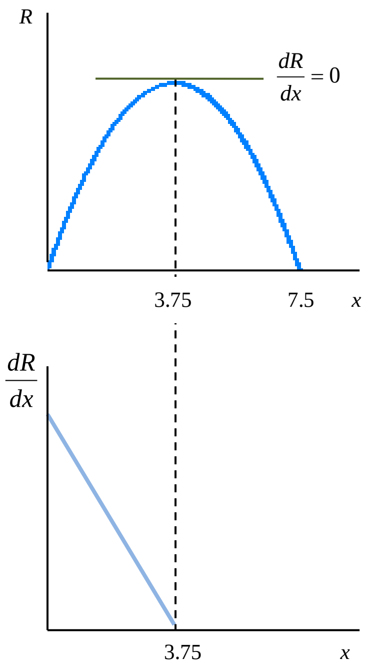
The slope of the total cost curve at this point is 20.

The slope of the total cost curve at any value of q is given by the marginal cost curve. For example, when q = 10, the slope of the total cost curve is 20.

3.2 Let R represent total revenue. $R = px = (6 - 0.8x)x = 6x - 0.8x^2$

Marginal revenue: $\frac{dR}{dx} = 6 - 1.6x$

The total and marginal revenue curves are shown below.



For values of $x < 3.75$ the slope of R is positive. As x increases the slope declines until $x = 3.75$ when it is zero.

$\frac{dR}{dx}$ declines towards zero as $x \to 3.75$ from the left side

3.3 The marginal propensity to consume is given by $\frac{dC}{dY}$.

$$\frac{dC}{dY} = 0.6 + 2(0.5)Y^{-0.5} = 0.6 + \frac{1}{Y^{0.5}}$$

To determine how the marginal propensity to consume changes as Y increases, take the derivative of this function and determine its sign.

$$\frac{d}{dY}\left(\frac{dC}{dY}\right) = \frac{d^2C}{dY^2} = -0.5Y^{-1.5} = \frac{-0.5}{Y^{1.5}} < 0 \text{ for } Y > 0$$

The marginal propensity to consume falls as income increases.

3.4 Marginal revenue is given by $\frac{dR}{dq}$.

$$R = 10\sqrt{2q+5} = 10(2q+5)^{\frac{1}{2}}$$

$$\frac{dR}{dq} = 10(\tfrac{1}{2})(2q+5)^{-\frac{1}{2}}(2) = \frac{10}{(2q+5)^{\frac{1}{2}}}$$

when $q = 2$:

$$\frac{dR}{dq} = \frac{10}{\sqrt{4+5}} = \frac{10}{3} = 3\tfrac{1}{3}$$

3.5 (i) The marginal product of labour is given by $\frac{dQ}{dL} = 120L - 3L^2$.

(ii) To determine how the marginal product is changing take the derivative:

$$\frac{d}{dL}\left(\frac{dQ}{dL}\right) = \frac{d^2Q}{dL^2} = 120 - 6L$$

The law of diminishing marginal product is operating if $\frac{d^2Q}{dL^2} < 0$, that is, if $120 - 6L < 0$. The values of L that satisfy this inequality can be determined by finding equivalent inequalities.

Add $6L$ to both sides:

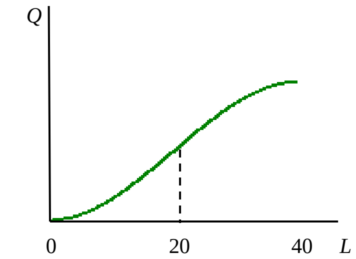
$$120 < 6L$$

Multiply both sides by $\frac{1}{6}$:

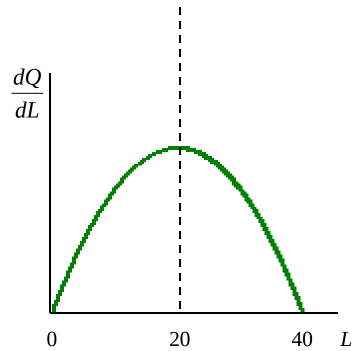
$$(\tfrac{1}{6})120 < (\tfrac{1}{6})6L \quad \Rightarrow \quad 20 < L$$

The solution set for this inequality, that is, the values of L for which this inequality holds, is $\{L \mid L > 20\}$. The law of diminishing marginal product, therefore, is operating when labour input is more than 20.

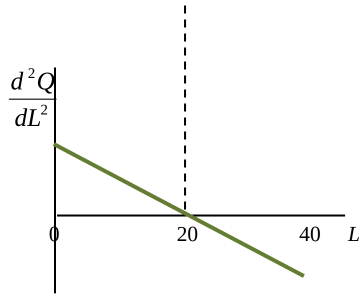
Representing $q(L)$, $\frac{dQ}{dL}$ and $\frac{d^2Q}{dL^2}$ graphically:



For values of $L < 20$, the slope of the total product function is increasing. At $L = 20$ there is a point of inflection and so the slope stops increasing. At this point the function changes from being convex to concave. Beyond it, the slope starts to fall but remains positive. It thus reaches its maximum value at $L = 20$.



The slope of the total product function is positive for $0 < L < 40$ with its maximum value at $L = 20$.



For $0 < L < 20$ the slope of the marginal product function declines towards zero. For $L > 20$, the slope of the marginal product function becomes negative.