(mathematics) inEconomics

Mathematical Methods in Economics: Problems and Solutions

Chapter 11

Worked Solutions to Problems on Equations and Functions II

2.1 (i) For equilibrium: $q_d = q_s$

$$\frac{8}{p^{\frac{1}{2}}} = p$$

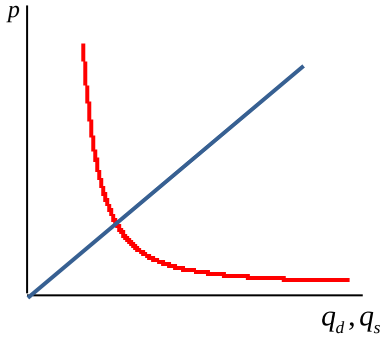
$$p^{\frac{3}{2}} = 8$$

$$p = 8^{\frac{2}{3}}$$

$$p = 4$$

When $p = 4$, from the supply function $q = 4$.

(ii)



(iii) Let S represent the lump-sum subsidy so the price paid by consumers is $p - S$. The demand function incorporating the subsidy is:

$$q_d = \frac{8}{(p-S)^{\frac{1}{2}}}$$

Following the payment of the subsidy, the equilibrium quantity is 10 and so, from the supply function, the equilibrium price is also 10. Substituting $q_d = 10$ and $p = 10$ into the demand function:

$$10 = \frac{8}{(10-S)^{\frac{1}{2}}}$$

Solving for S:

$$(10-S)^{\frac{1}{2}} = 0.8$$

$$(10-S) = (0.8)^2$$

$$S = 9.32$$

2.2 (i) $C = aq^{\alpha}$ so the following two equations must hold:

$$20 = a2^{\alpha} \qquad (1)$$

$$80 = a4^{\alpha} \qquad (2)$$

This is a set of *non-linear* simultaneous equations. Both equations are exponential equations so one way to solve them is by using logarithms[1].

$$\ln 20 = \ln a + \alpha \ln 2 \qquad (1a)$$

$$\ln 80 = \ln a + \alpha \ln 4 \qquad (2a)$$

(2a) – (1a)

$$\ln 80 - \ln 20 = \alpha \ln 4 - \alpha \ln 2$$

$$\ln\left(\frac{80}{20}\right) = \alpha(\ln 4 - \ln 2)$$

$$\ln 4 = \alpha \ln 2$$

$$\alpha = \frac{\ln 4}{\ln 2} = 2$$

when $\alpha = 2$:

$$20 = a2^2$$

$$a = 5$$

The equation of the total variable cost function is: $C = 5q^2$

(ii) Total fixed cost $= 40$

Total cost = total variable cost + total fixed cost $= 5q^2 + 40$

Average fixed cost $= \dfrac{\text{total fixed cost}}{q} = \dfrac{40}{q}$

Average variable cost $= \dfrac{\text{total variable cost}}{q} = \dfrac{5q^2}{q} = 5q$

Average total cost $= \dfrac{\text{total cost}}{q} = \dfrac{5q^2 + 40}{q} = 5q + \dfrac{40}{q}$

These are all rational[2] functions.

2.3 (i) Let $q_d = q_s = q$.

For equilibrium: $(19 - p)^{\frac{1}{2}} = -\frac{7}{2} + \frac{5}{2}p$

$$19 - p = (-\tfrac{7}{2} + \tfrac{5}{2}p)^2$$

Solving this quadratic equation gives $p = 3$ or $p = -0.36$. When $p = 3$ from the supply function $q = -\frac{7}{2} + \frac{5}{2}(3) = 4$.

(ii) A demand function with a constant elasticity of unity takes the form: $q_d = \dfrac{a}{p}$. Since the point $p = 3$, $q_d = 4$ must lie on this function, $a = 12$.

The equation of the new demand function is: $q_d = \dfrac{12}{p}$

(iii) If a proportionate tax at a rate t is imposed on suppliers:

$$q_s = -\tfrac{7}{2} + \tfrac{5}{2}(p - tp) = -\tfrac{7}{2} + \tfrac{5}{2}(1 - t)p$$

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For equilibrium:

$$\frac{12}{p} = -\tfrac{7}{2} + \tfrac{5}{2}(1-t)p$$

$$\tfrac{5}{2}(1-t)p^2 - \tfrac{7}{2}p - 12 = 0$$

Solving this quadratic equation:

$$p = \frac{-\left(-\tfrac{7}{2}\right) \pm \sqrt{\left(-\tfrac{7}{2}\right)^2 - 4\left(\tfrac{5}{2}(1-t)\right)(-12)}}{2\left(\tfrac{5}{2}(1-t)\right)} = \frac{\tfrac{7}{2} \pm \sqrt{\tfrac{529}{4} - 120t}}{5(1-t)}$$

To be economically meaningful the positive root gives the solution.

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