(mathematics) √ inEconomics

Mathematical Methods in Economics: Problems and Solutions

Chapter 10

Worked Solutions to Problems on Equations and Functions I

1.1 (i)

p	q_d
0	$q_d = 48 - 4(0) = 48$
3	$q_d = 48 - 4(3) = 36$
6	$q_d = 48 - 4(6) = 24$
8	$q_d = 48 - 4(8) = 16$
12	$q_d = 48 - 4(12) = 0$

(ii) Let E represent total expenditure.

p	q_d	$E = pq_d$
0	48	0
3	36	108
6	24	144
8	16	128
12	0	0

(iii) One way to rearrange the demand equation to express p in terms of q_d is as follows.

$$q_d = 48 - 4p$$

Subtract 48 from both sides:

$$q_d - 48 = 48 - 4p - 48$$

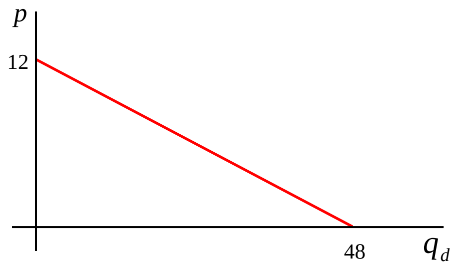
$$q_d - 48 = -4p$$

Divide both sides by -4:

$$\frac{q_d - 48}{-4} = \frac{-4p}{-4}$$

$$-\tfrac{1}{4}q_d + 12 = p$$

The inverse demand equation is $p = 12 - \frac{1}{4}q_d$. Its graph is shown in the diagram below.



This graph is called a demand curve.

1.2 (i) $f(p)$ is notation to indicate q_d is a function of p. Since $f(p)$ indicates the function contains only one independent variable, p, the remaining symbols in the expression on the right-hand side of the equation, a and b, must be parameters. Including it when defining the function therefore clarifies what type of quantity each symbol on the right-hand side of the equation represents. There are other ways of providing this information, for example by giving a verbal description of each symbol in the equation but $f(p)$ gives it in a concise and unambiguous way. In question 1.1 the equation that defines the demand function contains numerical constants and only one unknown p. This means p must be the independent variable so the form of the equation makes it unnecessary to include $f(p)$. However this notation could be, and often is, included when defining functions like that in question 1.1.

(ii)

$f(p) = a + bp$
$f(0) = a + b(0) = a$ $f(4) = a + b(4) = a + b4$ $f(d) = a + b(d) = a + bd$

(iii) One way to rearrange the supply equation to express p in terms of q_s is as follows.

$$q_s = a + bp$$

Subtract a from both sides:

$$q_s - a = a + bp - a$$

$$q_s - a = bp$$

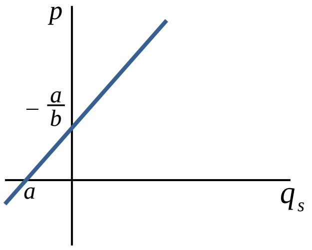
Divide both sides by b:

$$\frac{q_s - a}{b} = \frac{bp}{b}$$

$$\frac{1}{b}q_s - \frac{a}{b} = p$$

The equation that defines the inverse supply function is $p = -\frac{a}{b} + \frac{1}{b}q_s$. Using function notation the inverse supply function can be written $p = f^{-1}(q_s) = -\frac{a}{b} + \frac{1}{b}q_s$

(iv) The inverse supply equation is a linear. If $a < 0$ and $b > 0$ the intercept $\left(-\frac{a}{b}\right)$ will be positive since $-a > 0$. Since $b > 0$ the graph will be upward sloping.



This graph is called a supply curve.