(mathematics) √ inEconomics

Mathematical Methods in Economics: Problems and Solutions

Answers

Chapter 1

Equations and functions I

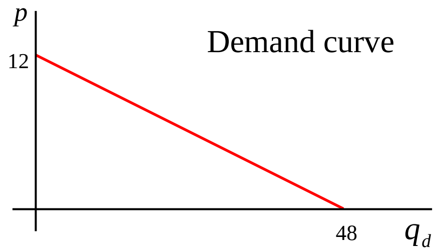
1.1 (i)

p	q_d
0	48
3	36
6	24
8	16
12	0

(ii)

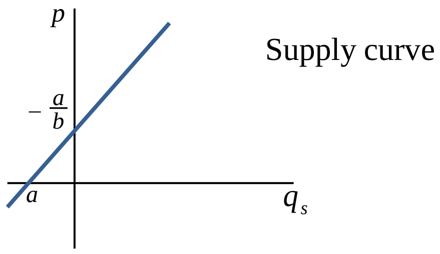
p	Expenditure
0	0
3	108
6	144
8	128
12	0

(iii) $p = 12 - \frac{1}{4} q_d$



1.2 (ii) $f(0) = a$, $f(4) = a + b4$, $f(d) = a + bd$ (iii) $p = -\frac{a}{b} + \frac{1}{b} q_s$

(iv)



1.3 (i)

Y	C
200	260
201	260.8
250	300
275	320
336	368.8
480	484

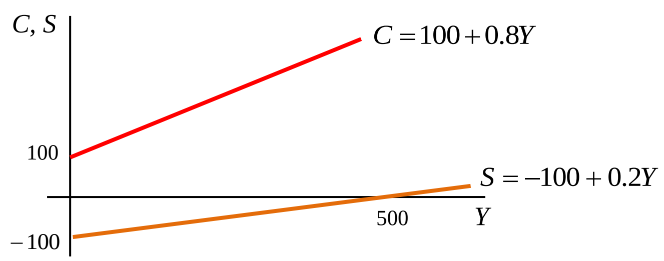
(ii)

$c(0) = 100$
$c(120) = 196$
$c(345) = 376$
$c(m) = 100 + 0.8m$

$c(0)$ is consumers' expenditure when income is zero.

(iii) $S = -100 + 0.2Y$

(iv)



(v)

Y	C	$\frac{\Delta C}{\Delta Y}$
200	260	
		0.8
201	260.8	
		0.8
250	300	
		0.8
275	320	
		0.8
336	368.8	
		0.8
480	484	

$\frac{\Delta C}{\Delta Y}$ is the marginal propensity to consume defined in terms of differences.

1.4 (i)

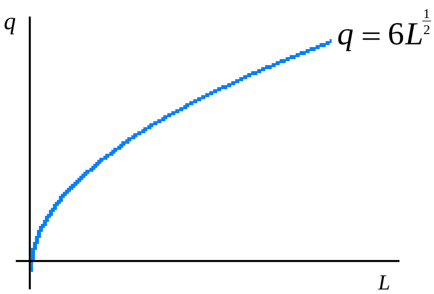
L	0	1	4	9	25	100
q	0	6	12	18	20	60

(ii)

L	q	$\frac{\Delta q}{\Delta L}$
0	0	
		6
1	6	
		2
4	12	
		1.2
9	18	
		0.75
25	30	
		0.4
100	60	

$\frac{\Delta q}{\Delta L}$ is the marginal product of labour defined in terms of differences.

(iii)



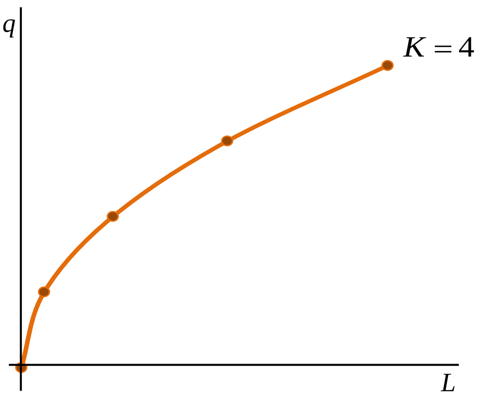
This graph is called the total product curve.

1.5 (i)

L	0	1	4	9	16	18	36	27
K	4	4	4	4	4	8	9	12
q	0	40	80	120	160	240	360	360

(ii) $f(0,0) = 0$, $f(7,28) = 280$, $f(a,b) = 20(ab)^{0.5}$, $f(L_0, K_0) = 20L_0^{0.5}K_0^{0.5}$

(iii)



Total product of labour curve

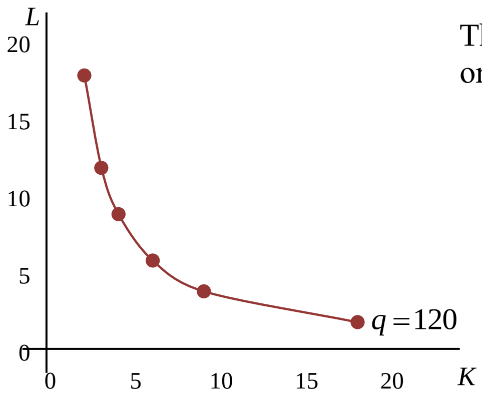
(iv) $L = \dfrac{36}{K}$

.

(v) Six possible combinations are:

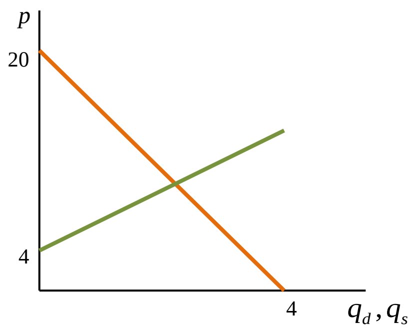
2	3	4	6	9	18
18	12	9	6	4	2

(vi)



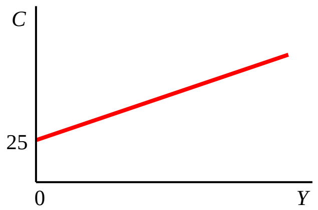
This graph is called an isoquant or 'production' isoquant.

1.6 (i)



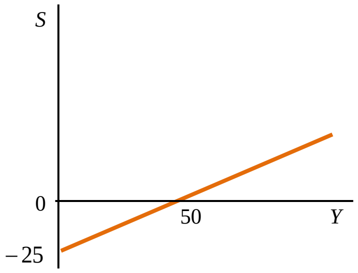
(ii) $p = 10$, $q_d = q_s = 2$ (iii) $q = 3$ (iv) No

1.7 (i) $C = 25 + \frac{1}{2}Y$

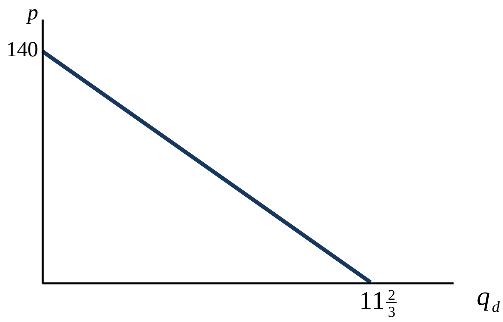


(ii) $C = 325$

(iii) $S = -25 + \frac{1}{2}Y$

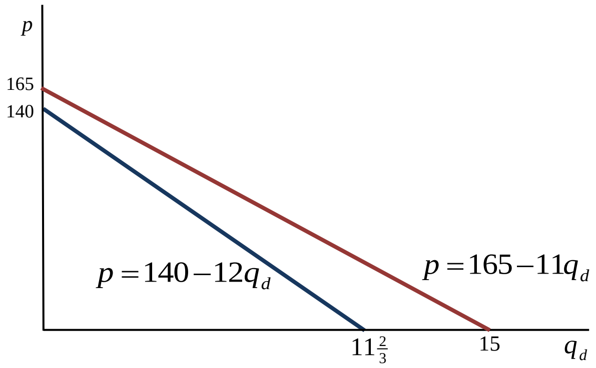


1.8 (i)



(ii) $q_d = 4$

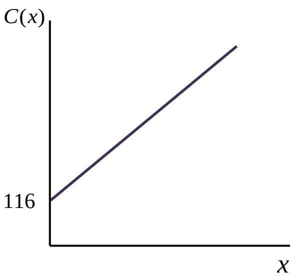
(iii)

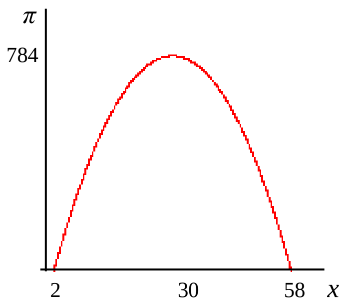


$p = 121, \; p = 78 + \frac{7}{2}q_s$

1.9 (i) $p = 6$ (ii) $33\frac{1}{3}\%$

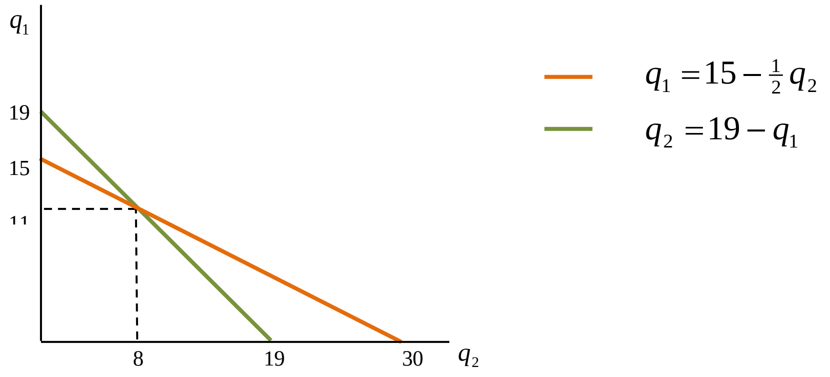
1.10 (i) $C(x) = 116 + 30x$ (ii) $\pi = 60x - x^2 - 116$ (iii) $x = 30$





1.11 $r \cong 0.0744$

1.12 $q_1 = 11$ and $q_2 = 8$



1.13 *a.* $Y = \dfrac{a + \bar{I} + \bar{G} - bT}{1-b}$, $C = a + b(Y - T)$, $R_G = T$

b. $Y = \dfrac{a + \bar{I} + \bar{G}}{1-b+bt}$, $C = \dfrac{a + b(1-t)\bar{I} + b(1-t)\bar{G}}{1-b+bt}$, $R_G = T = tY = t\left[\dfrac{a + \bar{I} + \bar{G}}{1-b+bt}\right]$

1. The equilibrium level of income is reduced:

Without taxation: $Y = \dfrac{a + \bar{I} + \bar{G}}{1-b}$

With lump-sum taxation: $Y = \dfrac{a + \bar{I} + \bar{G} - T}{1-b}$

With proportional taxation: $Y = \dfrac{a + \bar{I} + \bar{G}}{1-b+bt}$

2. With proportional taxation the value of the multiplier (m) is also reduced thus reducing the effect of a change in autonomous expenditure.

Without taxation: $m = \dfrac{1}{1-b}$

With lump-sum taxation: $m = \dfrac{1}{1-b}$

With proportional taxation: $m = \dfrac{1}{1-b+bt}$

1.14 (i) $\lim_{q_s \to 0^+} s(q_s) = 12.5$, $\lim_{q_s \to \infty} s(q_s) = 80$ (ii) $p = 26$, $q = 3$

1.15 (i) $p = 6$, $q = 2$ (ii) $T = 11$

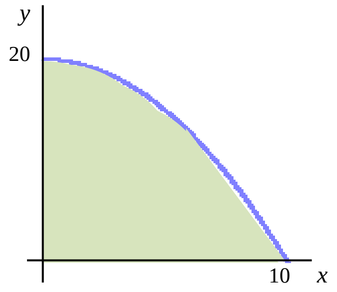
1.16 $x = 72$, $y = 1$ and $x = 2$, $y = 36$

1.17 $q_1 = 15$, $q_2 = 50$

1.18 (i) $p = 64$, $q = 4$

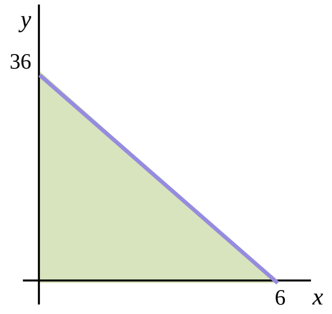
1.19 (i) Country A: $x = 10$, $y = 20$ Country B: $x = 6$, $y = 36$

(ii) Country A:



Production possibility set – all points in the area shaded green or lying along the production possibility frontier

Country B:



Production possibility set – all points in the area shaded green or lying along the production possibility frontier

(iii) Country A: 2.2 units of food Country B: 6 units of food

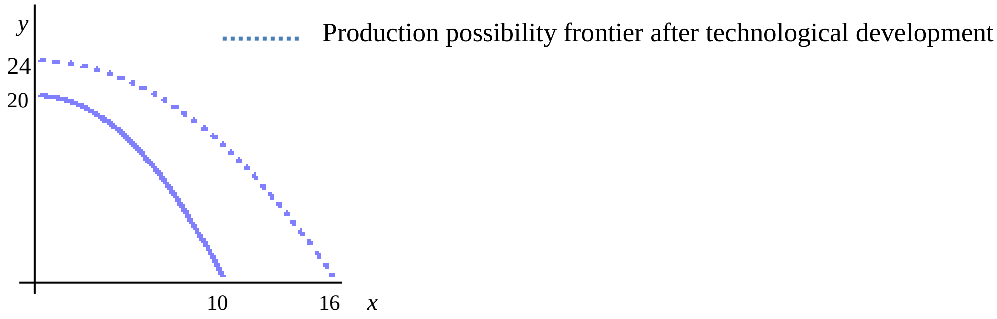
Opportunity costs are increasing in A and constant in B

(iv) Resources are not used efficiently

(v) Yes

(vi) For example: Country A: $x = 8$, $y = 7.2$ Country B: $x = 1$, $y = 3$

(vii) $y = 24 - \frac{3}{32}x^2$



(viii) The production possibility frontier is the consumption possibility frontier.

1.20 $C = f(Y) = -\frac{1}{100}Y^2 + \frac{8}{10}Y$

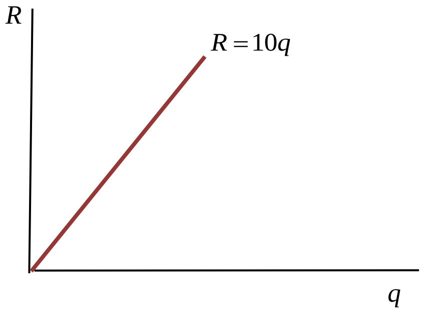
1.21 (i) $\alpha = 0.5$ (ii) Two possible combinations: $L = 4$, $K = 50$ and $L = 25$, $K = 20$ (iii) $q^* = 14$

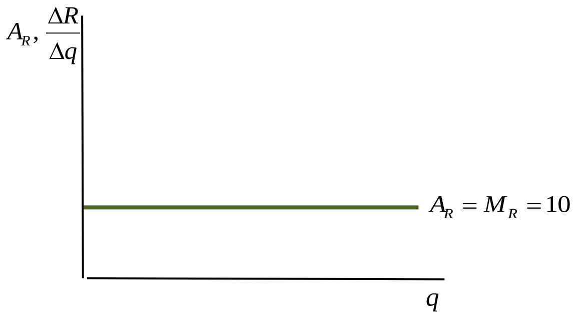
1.22 (i) $x = 4$ (ii) $x = 116$, $y = 1$ (iii) One of an infinite set of bundles is $x = 20$ and $y = 25$

1.23 (i) $c(0) = 80$, $d(0) = 200$ (ii) $r(x) = 200x - 0.5x^2$

1.24 *a*. (i) $R = pq = 10q$, $A_R = \frac{R}{q} = \frac{10q}{q} = 10$, $M_R = \frac{\Delta R}{\Delta q} = 10$

(ii)





b. (i) $R = pq = (40 - 2q)q = 40q - 2q^2$, $A_R = \frac{R}{q} = \frac{40q - 2q^2}{q} = 40 - 2q$

(ii) R is defined by a quadratic equation and A_R by a linear equation.

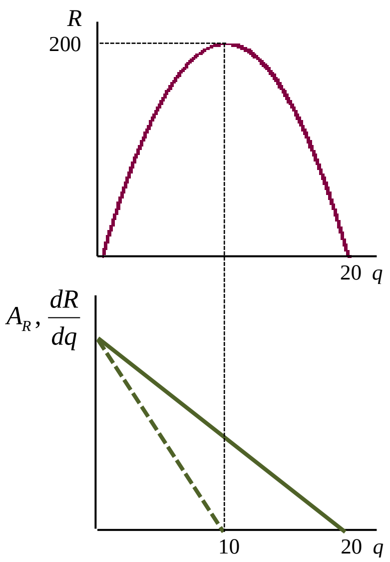
(iii) Both are linear and have the same *y*-intercept. The marginal revenue curve is twice as steep as the average revenue curve.

(iv)

(ii) R is defined by a quadratic equation and A_R by a linear equation.

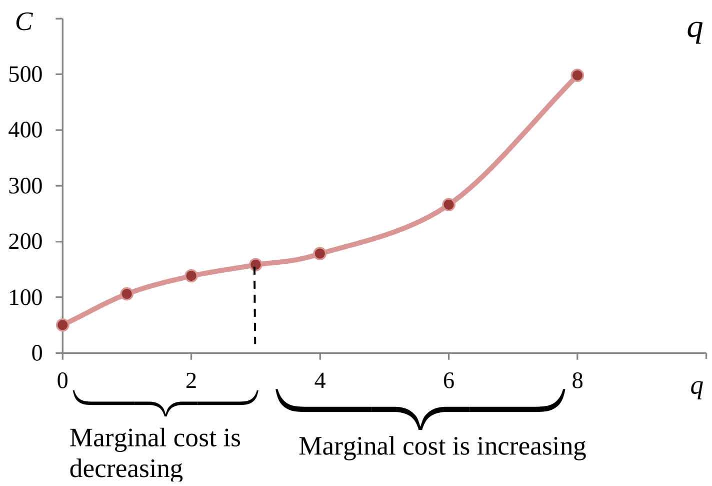
(iii) Both are linear and have the same *y*-intercept. The marginal revenue curve is twice as steep as the average revenue curve.

(iv)



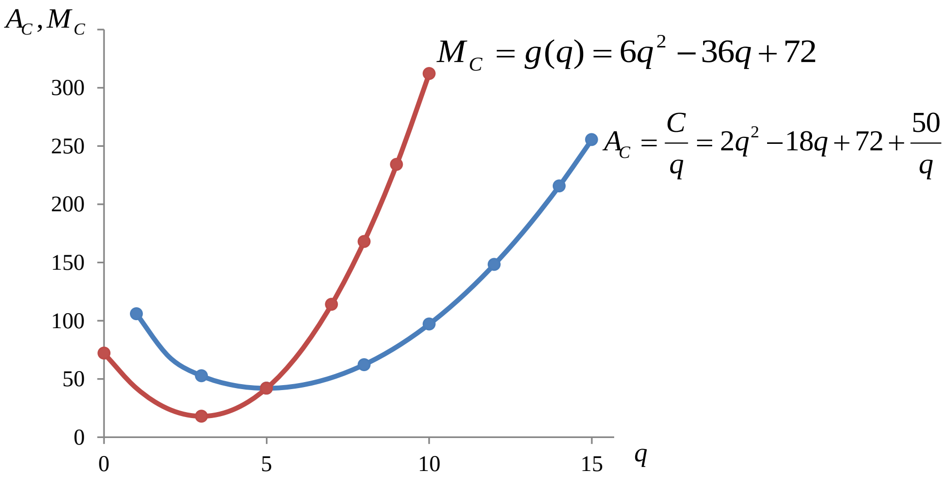
(v) $q = 10$, $M_R = 0$ (vi) $q = 0$ and $q = 20$

1.25 (i)



$q = 5,\ A_c = 42,\ M_C = 42$

(ii)



1.26 (i) $p = 3$, $q = 39$, consumer surplus $= 380.25$

(ii) $p = 6$, $q = 33$, Δ consumer surplus $= -108$

(iii) Consumers' share is $\frac{12}{13}$, Producers' share is $\frac{1}{13}$

1.27 (i) $x = 4$ and $y = 20$ (ii) $x = 5$, price has fallen by £1.8.

1.28 $q_1 = 8$, $q_2 = 3$, $q_3 = 5$, $p = 137.2$

1.29 $q_1 = 48 - 0.4q_2$, $q_2 = 40 - 0.5q_1$.

1.30 (i) $Q = 13{,}500{,}000$ (ii) $Q = 14{,}435{,}471$

(iii) In the seventeenth month, output will be 20% above its current level.

(iv) $r \cong 0.039$ or 3.9%

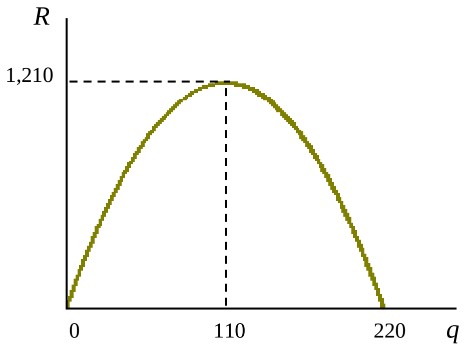
1.31 *a*. The tariff is reduced to £3 per unit:

Δ consumer surplus = 348, Δ producer surplus = –306, Δ total surplus = 42 *b*. The tariff is abolished:

Δ consumer surplus = 768, Δ producer surplus = –600, Δ total surplus = 168

1.32 (i) $q = 220 - 10p$

(ii)



$q = 110,\ R = 1{,}210$

(iii) q = 12.5 or q = 120

1.33 (i) $R = 160q - 5q^2$ (ii) $q = 32 - 0.2p$ (iii) $p = 80$

1.34 $q = 256$

1.35 (i) Consumer surplus is 1,638.4, Producer surplus is 2,457.6 (ii) The tax imposed is 4.

(iii) $R = 480$, Consumers' share is 40%. Producers' share is 60%.

1.36 (i) $p = 6$, $q = 14$ (ii) Total revenue to producers is £176.

(iii) *a*. The lump-sum subsidy is 4. *b*. The proportionate subsidy is 0.5.

(iv) Adopt one of the subsidies.

1.37 (i) $p = \frac{5}{4}y$, $q = 9y$. (ii) When $y = 4, \begin{Bmatrix} p = 5 \\ q = 36 \end{Bmatrix}$, when $y = 8, \begin{Bmatrix} p = 10 \\ q = 72 \end{Bmatrix}$. (iii) Yes.

1.38 (i) $Y = 4{,}000$ (ii) $Y = 4{,}600$ (iii) $Y = 4{,}200$ (iv) $t = \dfrac{1}{21}$

1.39 (i) $p = 50 + 30q$, The supply function (ii) $p = 160 - \frac{2}{3}q$, The demand function

(iii) $p = 20 + 4q$, Not an economic function.

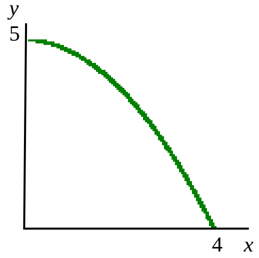
1.40 $q_d = 60$

1.41 (i) $p = 10$, $q = 2{,}000$

(ii) 1. A subsidy of 1.8. 2. The government must purchase 450 tonnes. 3. The subsidy.

1.42 (i) $y = 5$ (ii) $x = 4$ (iii) $x = 2$, $y = 3$

(iv)

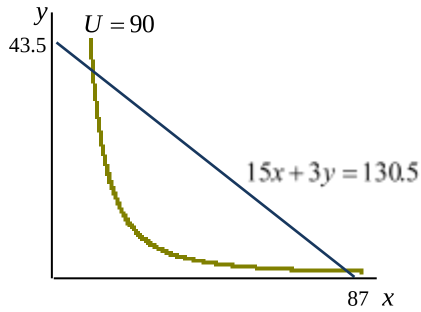


(v) Efficient use of resources, $(2,3)$, unemployment, $(1,3)$, unattainable, $(3,2)$

1.43 (i) $Y = \dfrac{\alpha + \bar{I} + \bar{G}}{1-\beta+\beta t}$ (ii) $m = \dfrac{1}{1-\beta+\beta t}$ (iii) $a.\ C = 180 + 0.8Y_d$ $b.\ \Delta Y = 120$

1.44 (i) $\alpha = 0.5$

(ii) $1.5x + 3y = I$

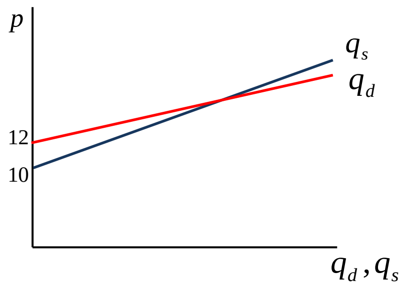


Yes

(iii) $x = 16,\ p_x = 2$

(iv) One of an infinite number of bundles is $x = 56,\ y = 4$. An income of 124.

1.45 (i)



$p = 16,\ q = 16$

(ii) $p = 22,\ q = 32$

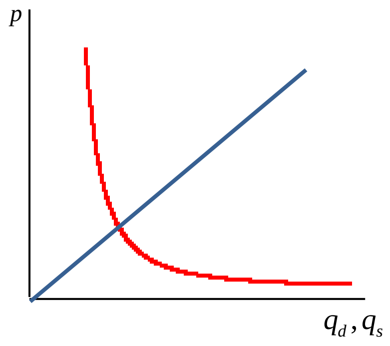
(iii) Before the shift in the demand curve the market is unstable, after the shift it is stable.

Chapter 2

Equations and Functions II

2.1 (i) $p = 4$, $q = 4$.

(ii)



(iii) A subsidy of 9.32.

2.2 (i) $C = 5q^2$

(ii) Average fixed cost = $\dfrac{40}{q}$, Average variable cost = $5q$, Average total cost = $5q + \dfrac{40}{q}$,

Rational functions

2.3 (i) $p = 3$, $q = 4$ (ii) $q_d = \dfrac{12}{p}$ (iii) $p = \dfrac{\frac{7}{2} + \sqrt{\frac{529}{4} - 120t}}{5(1-t)}$

2.4 (i) $x_1 = 20$ (ii) No

(iii) Three possibilities: $x_1 = 16$, $x_2 = 15{,}616$, $x_1 = 14$, $x_2 = 21{,}024$, $x_1 = 12$, $x_2 = 25{,}088$

(iv) $a = 13{,}824$, $b = 0.4$

2,5 (i) $p = \dfrac{\gamma + \alpha}{\beta + \delta} + \dfrac{\delta T}{\beta + \delta}$, $q = \dfrac{\alpha\delta - \beta\gamma}{\beta + \delta} - \dfrac{\beta\delta T}{\beta + \delta}$ (ii) $\Delta p = \dfrac{\delta}{\beta + \delta}\Delta T$, $\Delta q = -\dfrac{\beta\delta}{\beta + \delta}\Delta T$

2.6 (i) $r = 0.02$ (ii) 175.47 tonnes

2.7 (i) $p = 11$, $q = 192$ (ii) $q_d = 110 - 4p$, an inferior good

2.8 *a.* $Y = 1{,}362$, budget deficit *b.* $Y = 1312.5$, budget surplus *c.* $Y = 1{,}245$, budget surplus

2.9 (i) $y = 20$ (ii) $x = 10$ (iii) $x = 5, y = 15$ (iv) *a.* $x = 5$, $y = 14$ *b.* $x = 5$, $y = 16$

(v) $x = 12.95$ (vi) $y = 24.5 - 0.15x$.

2.10 (i) $Y = 1{,}600$ (ii) Increase G by 256, reduce the tax rate to 2.4%. Policy A (iii) $G = 485\frac{1}{3}$

2.11 (i) $p = 2$ (ii) % change in price $= 50\%$, % change in quantity $= 100\%$

2.12 (i) $y = 104 - 8x$, $x = 8$ and $y = 40$ (ii) $y = 90 - 6x$

2.13 (i) $Y = 3{,}600$, $C = 2{,}360$ (ii) $Y = 4{,}200$ (iii) Reduce G by 490, increase tax rate to 23.7%

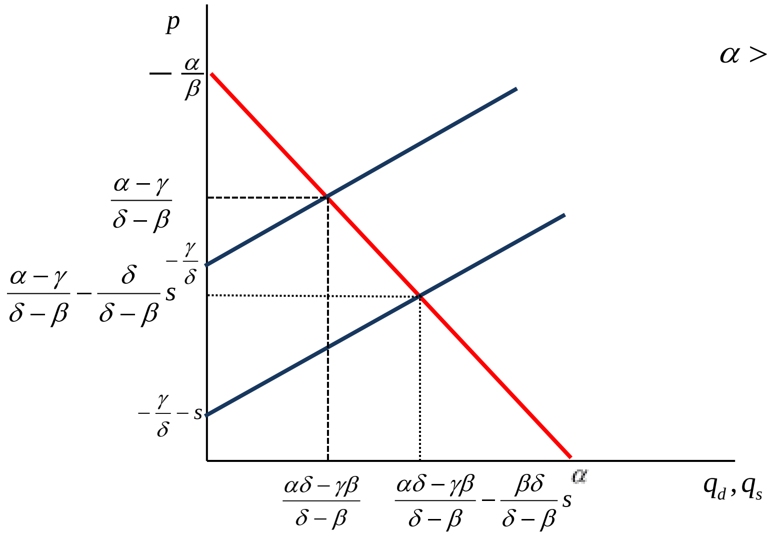
2.14 (i) $p = 148$, $q = 36$ (ii) 216 (iii) $p = 214$, $q = 81$ (iv) Δ change in consumer surplus = 877.5

2.15 (i) $p = \dfrac{\alpha - \gamma}{\delta - \beta} - \dfrac{\delta s}{\delta - \beta}$, $\quad q = \dfrac{\alpha\delta - \gamma\beta}{\delta - \beta} - \dfrac{\beta\delta s}{\delta - \beta}$

(ii)

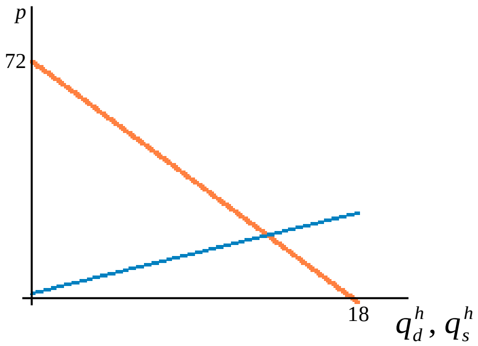
Assuming this is a market for a normal good.

$\alpha > 0,\ \delta > 0,\ \beta < 0,\ \gamma < 0$



(iii) $C = \left[\dfrac{\alpha\delta - \gamma\beta}{\delta - \beta} - \dfrac{\beta\delta s}{\delta - \beta}\right]s$

2.16 (i)



The degree of import penetration is 0.75.

(ii) *a*. Revenue to the government is £36. *b*. The deadweight loss to society is 18

(iii) *a*. £12 *b*. $T = 5\frac{1}{3}$ or $T = 64$.

c. No domestic production, 13 tonnes would be imported. A tariff of 60 is needed.

2.17 (i) $q = \dfrac{\alpha}{\beta(n+1)}$, $nq = \dfrac{n}{(n+1)}\left(\dfrac{\alpha}{\beta}\right)$

(ii) Output in the oligopolistic market is 60% above monopoly output.

$$\lim_{n\to\infty}\left(\frac{n\alpha}{(n+1)\beta}\right) = \frac{\alpha}{\beta}$$

2.18 (i) $p = 10.6$, $q = 26$ (ii) Total revenue = 275.6. (iii) Producer surplus $= 67.6$

(iv) The price ceiling is 10.2, producer surplus falls by 10.

2.19 (i) *a.* $Y = \dfrac{(a_0 - a_1 t_0) + \bar{I} + \bar{G}}{1 - a_1(1 - t_1)}$ *b.* $\dfrac{\Delta Y}{\Delta I} = \dfrac{1}{1 - a_1(1 - t_1)}$

(ii) *a.* $C = 148 + 0.84Y_d$ *b.* $C = 1{,}907.8$ *c.* $Y = 1{,}350$ *d.* $G = 255$

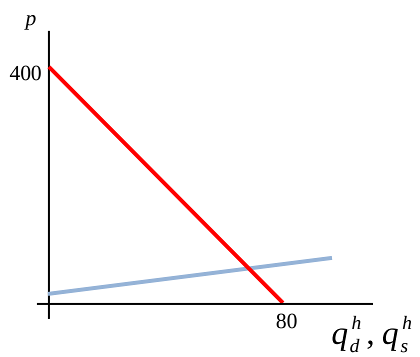
2.20 (i) $Y = \dfrac{a_0 + \bar{I} + \bar{G}}{1 - a_1(1 - t)}$, $C = a_0 + a_1(1 - t)Y = a_0 + (1 - t)\left(\dfrac{a_0 + \bar{I} + \bar{G}}{1 - a_1(1 - t)}\right)$

(ii) $Y = 8{,}000$

(iii) The government must increase expenditure by 720; a budget deficit of 220 will result.

The government must cut the tax rate to 11%.; a budget deficit of 400 will result.

2.21 (i)



(ii) 70.4 (iii) $\frac{1}{4}$

(iv)

		Free trade	Tariff	Subsidy
Total surplus		$14{,}551\frac{1}{9}$	$14{,}526\frac{1}{9}$	$14{,}481\frac{8}{9}$
Δ Total surplus relative to free trade			– 25	$-69\frac{2}{9}$
Consumer surplus		14,440	$14{,}062\frac{1}{2}$	14,440
Δ Consumer surplus relative to free			$-377\frac{1}{2}$	0
Producer surplus		$111\frac{1}{9}$	$233\frac{11}{18}$	$186\frac{8}{9}$
Δ Producer surplus relative to free trade			$122\frac{1}{2}$	$75\frac{7}{9}$
Government:	costs	0	0	145
	revenue	0	230	0

(v) The price and quantity traded in the domestic market would be the equilibrium values of $p = 48$ and $q = 70.4$. Producers would increase their surplus at the expense of consumers.

Consumer surplus: $12{,}390.4$, producer surplus: $1{,}376\frac{32}{45}$

2.22 (i) Above £2 (ii) q = 25 (iii) p = 4, q = 50 (iv) $p = 32 \cdot 2^{-0.02q}$

2.23 (i) $x = 5$ (ii) $y = 8$ (iii) $y = 6.4$ (iv) $x = 4, y = 4.8$

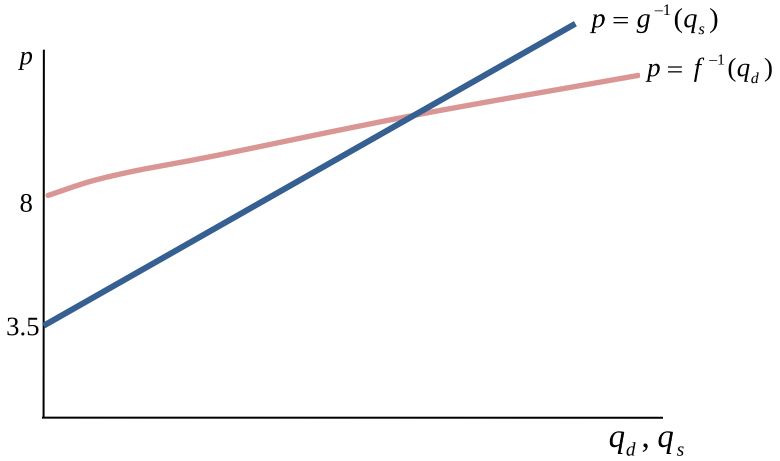
(v) At $x = 2.5$, $y = 6.928$ (vi) $\frac{x^2}{25} + \frac{y^2}{100} = 1$

2.24 (i) $p = 10, \quad q = 48$

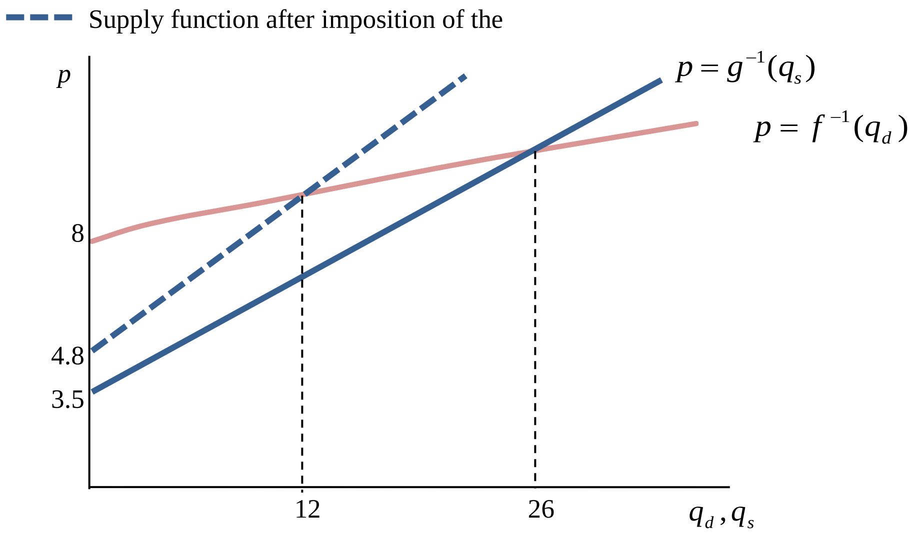
(ii) $a.\ \ p = 9.6$ $b.$ £1,255.176 $c.$ Δ consumer surplus $= -1{,}324.764$

2.25 (i) $p = 10$

(ii)



(iii) $p = 9, \ q = 12$



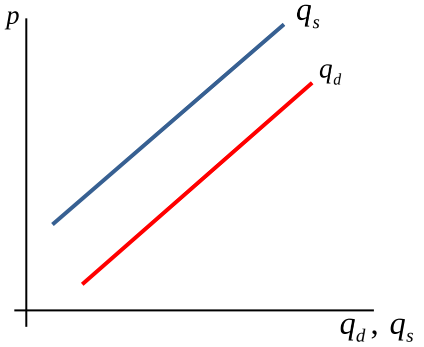
(iv) Before the imposition of the tax, the equilibrium market price is 10. After its imposition the equilibrium price is 9. However, after government intervention, at a price of 10 there is excess demand so There will be pressures on the market price to *increase* rather than fall to the new equilibrium of 9. The equilibrium in this market is (Walrasian) unstable.

2.26 (i) $Y = \dfrac{a + e + (d + f)r_0 + G_0}{1 - b}, \quad C = \dfrac{a + be + (bf + d)r_0 + bG_0}{1 - b}$

(ii) $C = 150 + 0.75Y - 0.6r$, $I = 3{,}000 - 4r$ (iii) $Y = 26{,}797.6$

2.27 (i) $p = \dfrac{a - \alpha}{b + \beta}$: $\quad q = a - b\left(\dfrac{a - \alpha}{b + \beta}\right) = \dfrac{a(b + \beta) - b(a - \alpha)}{b + \beta} = \dfrac{a\beta + b\alpha}{b + \beta}$ for $b + \beta \neq 0$

(ii) There is no unique solution to this model if $b+\beta=0 \quad \Rightarrow \quad b=-\beta$

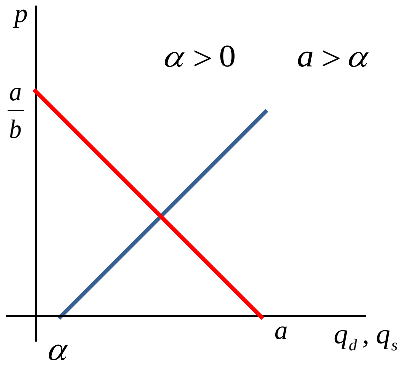


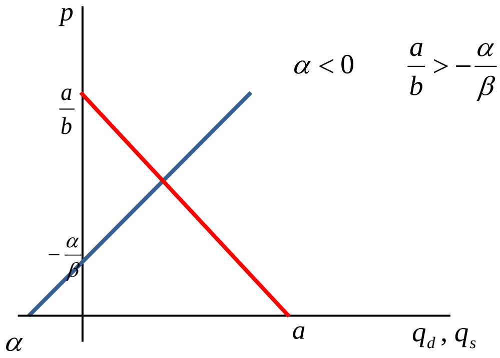
$\beta > 0$

$-\frac{\alpha}{\beta} > \frac{a}{b}$

If $b=-\beta$ and $a=\alpha$ then there is an infinite set of real values for p that satisfy this equation so the model has an infinite set of solutions. In this case the demand and supply functions are the same and so are represented graphically by the same line.

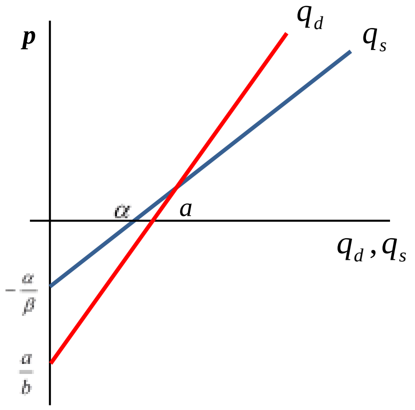
(iii) Normal good 1. $\alpha > 0$ and $a > \alpha$ 2. $\alpha < 0$ and $\frac{a}{b} > -\frac{\alpha}{\beta}$





	Constraints on the parameter values		
	Both require	$p > 0$	$q > 0$
$a.$	$-\frac{1}{b} > \frac{1}{\beta}$ The slope of the demand function is *greater* than the slope of the supply function.		
1	$a > 0, \quad \alpha > 0$	$a > \alpha$	$-\frac{\alpha}{\beta} > \frac{a}{b}$
2	$a > 0, \quad \alpha < 0$		
3	$a < 0, \quad \alpha < 0$	$a > \alpha$	$-\frac{\alpha}{\beta} > \frac{a}{b}$
$b.$	$-\frac{1}{b} < \frac{1}{\beta}$ The slope of the demand function is *less* than the slope of the supply function.		
4	$a > 0, \quad \alpha > 0$	$a < \alpha$	$-\frac{\alpha}{\beta} < \frac{a}{b}$
5	$a < 0, \quad \alpha > 0$		
6	$a < 0, \quad \alpha < 0$	$a < \alpha$	$-\frac{\alpha}{\beta} < \frac{a}{b}$

1.



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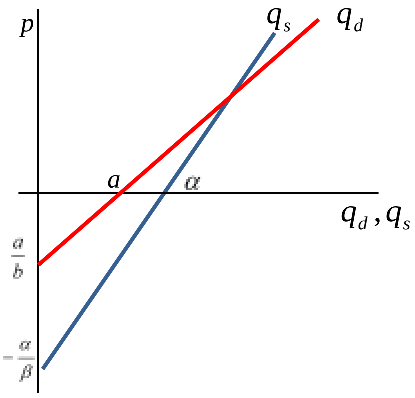

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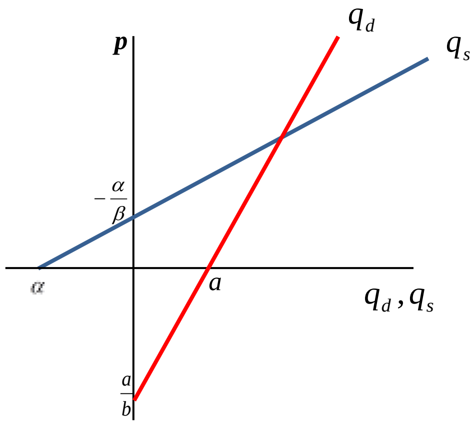

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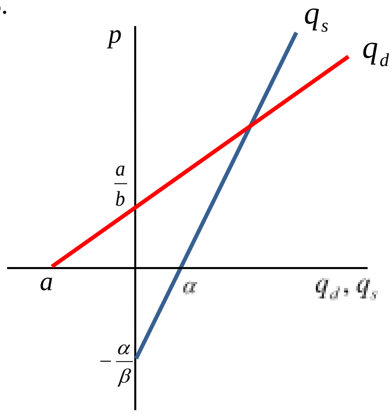
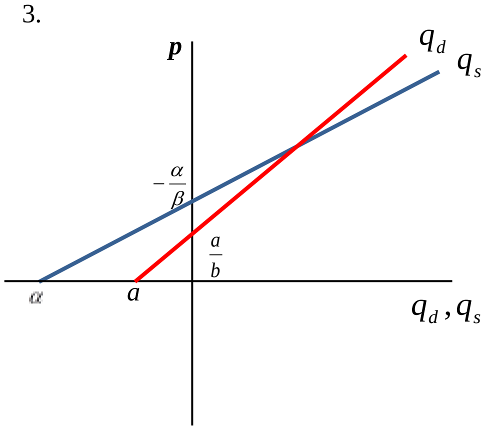


3.



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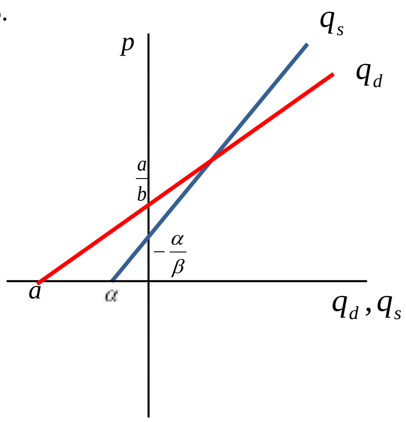


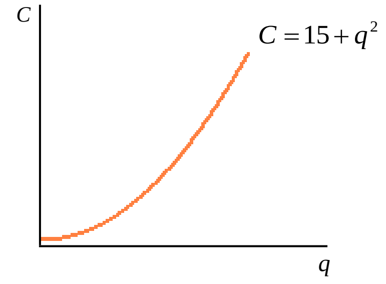
(iv) $p = \dfrac{-(b+\beta) \pm \sqrt{(b+\beta)^2 - 4(a-\alpha)(c-\gamma)}}{2(c-\gamma)}$, $(b+\beta)^2 \geq 4(c-\gamma)(a-\alpha)$

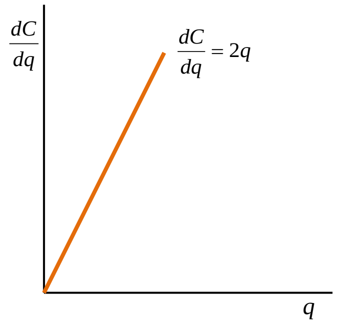
2.28 $p_1 = 20$, $q_1 = 50$, $p_2 = 112$ in (5) gives $q_2 = 24$.

Chapter 3

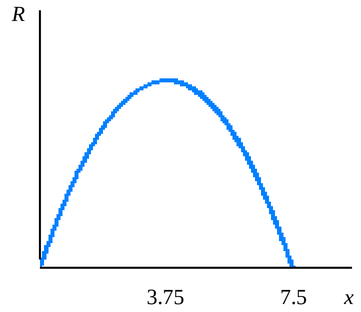
Differentiating a Function of One Variable

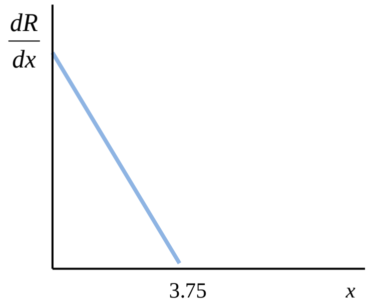
3.1 $\frac{dC}{dq} = 2q$





3.2 $R = 6x - 0.8x^2$, $\frac{dR}{dx} = 6 - 1.6x$

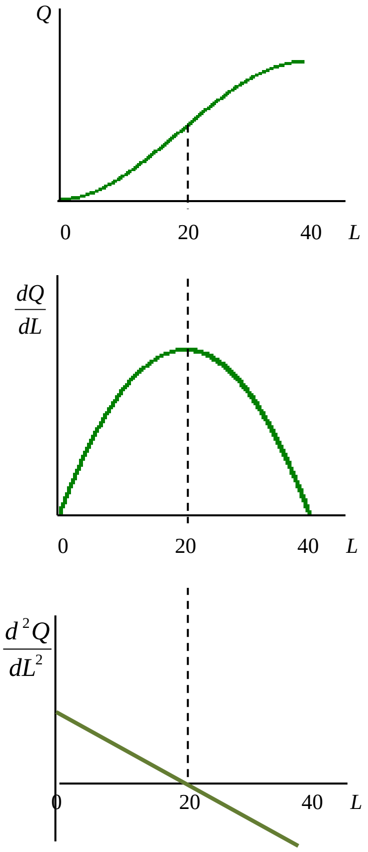




3.3 $\frac{dC}{dY} = 0.6 + \frac{1}{Y^{0.5}}$. The marginal propensity to consume falls as income increases.

3.4 $\frac{dR}{dq} = 3\tfrac{1}{3}$

3.5 (i) $\dfrac{dQ}{dL} = 120L - 3L^2$ (ii) $\{L | L > 20\}$



3.6 $\dfrac{dR(q)}{dq} = 3.875$

3.7 Let A = average fixed cost: $\dfrac{dA}{dq} = -\dfrac{F}{q^2}$

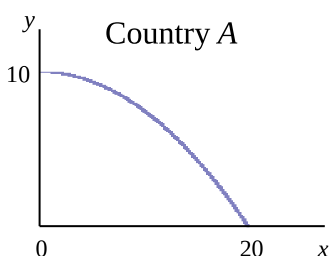
3.8 $\dfrac{C(x)}{x} = \dfrac{x^2}{10} - 47x + 8{,}000 + \dfrac{58{,}500}{x}$, $\dfrac{dC(x)}{dx} = \dfrac{3x^2}{10} - 94x + 8{,}000$, Average cost $= 3{,}105$,
Marginal cost $= 800$

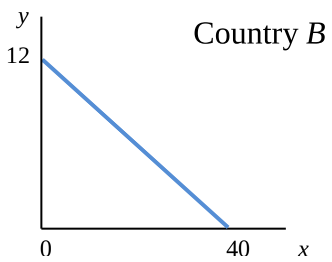
3.9 (i) $\dfrac{dQ}{dL} = a + 2bL$ (ii) The law of diminishing marginal product is not operating.

3.10 (i) $C = q^2 - 6q + 25$ (ii) *a.* $C = 305$ *b.* $C = 25$ (iii) $q = 1$ or $q = 25$.

(iv) Average fixed cost: $\dfrac{25}{q}$, Average variable cost: $q - 6$, Marginal cost: $\dfrac{dC}{dq} = 2q - 6$

3.11 (i)





(ii) Yes

(iii) Country *A*: opportunity cost is $0.5x$ so increasing, Country *B*: opportunity cost is 3 and so constant.

(iv) Country *A*: $y = 120 - \frac{5}{24}x^2$, Country *B*: $y = 180 - 3x$

Country *A*: $r \cong 0.0466$, Country *B*: $r \cong 0.1067$

Initially country *B* is producing relatively more capital goods than country *A*.

(v) $(42, 48)$ This production point represents a Pareto improvement because more of both goods is produced at this point.

3.12 Firm 1: q = 20, average cost = 143. Firm 2: q = 5, average cost = 176. No.

3.13 $\frac{dq}{dp} \cdot \frac{p}{q} = -\frac{p}{(p-100)} = -\frac{(q+6)}{6}$

3.14 $\eta = -2$

3.15 (i) Market 1: $p = 35 - \frac{1}{3}q$, Market 2: $p = 5 - \frac{1}{6}q$

(ii) Market 1: $\eta = \frac{-3p}{105 - 3p}$, Market 2: $\eta = \frac{-6p}{30 - 6p}$

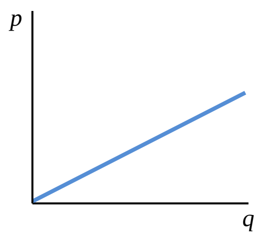
(iii) *a*. Market 1: q = 48, p = 19 Market 2: q = 6, p = 4

b. Market 1: $\eta = -1.1875$ Market 2: $\eta = -4$

c. Market 1: 768 Market 2: 24

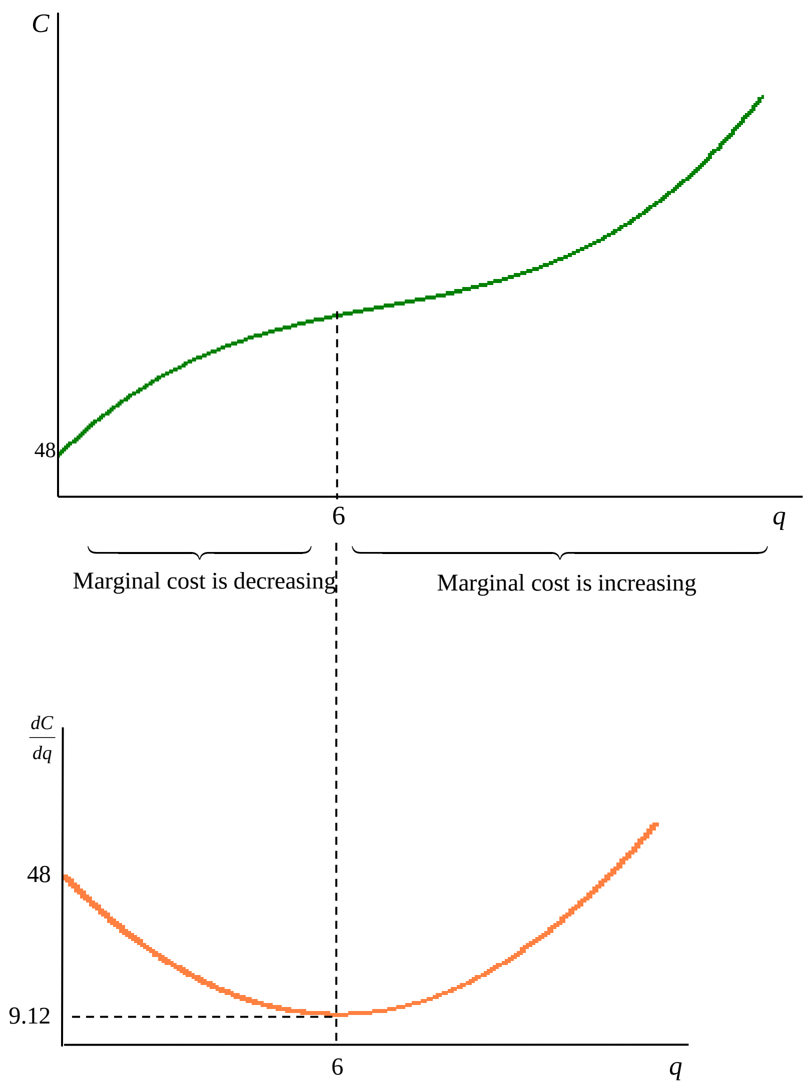
3.16 (i) $\eta = \frac{-4(11 - 0.25q)}{q}$ (ii) Marginal revenue = Marginal cost = 1

3.17 The elasticity of supply = 1.



3.18 (i) $\eta = \frac{-p}{1{,}000 - p}$ (ii) $p = 500$, $\frac{dR}{dq} = 0$

3.19 Marginal cost is decreasing for $q < 6$ and increasing for $q > 6$



3.20 (i) -2, -0.8 (ii) $h(p)$ is relatively less elastic at any price.

(iii) For $g(p)$ reduce price for $h(p)$ increase it.

3.21 (i) $\eta = -\frac{3}{2}\left(\frac{8}{15}\right) = -0.8$, $\varepsilon = 3\left(\frac{8}{15}\right) = 1.6$ (ii) Revenue to producers will be reduced.

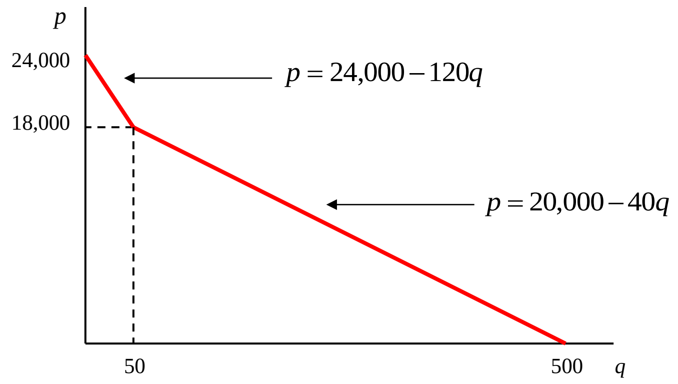
3.22 (i) $\eta = \frac{-p}{5-p}$ (ii) $\varepsilon = \frac{3p}{-5+3p}$ (iii) $\eta = -4$, $\varepsilon = \frac{12}{7}$ (iv) $p = \frac{5}{2}$

3.23 $\eta = -0.5p$, $p = 2$

3.24 (i) UK: $q = 300 - \frac{1}{60}p$, France: $q = 200 - \frac{1}{120}p$

(ii) $q = 500 - \frac{1}{40}p$ for $0 \le p \le 18{,}000$ (iii) 6,000

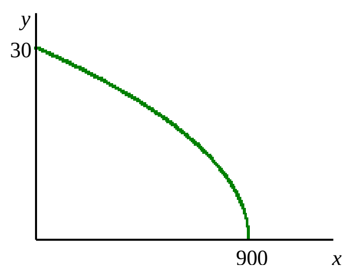
$q = 200 - \frac{1}{120}p$ for $18{,}000 \le p \le 24{,}000$



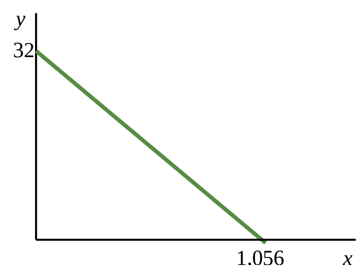
3.25 (i) $p = f(q) = 3e^{q}$ (ii) $q = \ln \frac{p}{3}$

3.26 (i)

Country *A*



Country *B*



(ii) Country *A*: $-\frac{dy}{dx} = \frac{1}{2(900-x)^{\frac{1}{2}}}$, increasing Country *B*: $-\frac{dy}{dx} = \frac{1}{33}$, constant

(iii) Country *A*: $y = 25$, $x = 275$ Country *B*: $y = 24$, $x = 11(24) = 264$

(iv) Country *A*: Yes Country *B*: No

(v) $a = 1{,}296$, $b = 1.125$

3.27 $\frac{dC}{dq} = 1.875q^{\frac{1}{3}}$

3.28 (i) $p = 72$, $q = 8$, the elasticity of supply = 1. At the new equilibrium the elasticity of supply will be unity.

(ii) $q = \frac{-9+\sqrt{1681+8s}}{4}$, $p = 9\left(\frac{-9+\sqrt{1681+8s}}{4}\right) - s$, 1.25%

3.29 (i) $\eta = -3$ (ii) A reduction in price will always result in an increase in revenue.

3.30 (i) $\eta = 1 - \frac{\alpha}{\beta q}$ (ii) $-\infty < \eta < 0$

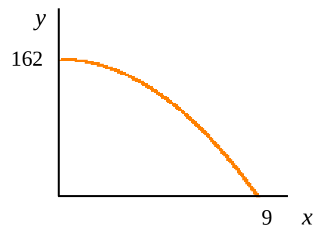
3.31 $\frac{dC}{dq} = \frac{w}{f'(L)}$

3.32 (i) $\frac{dY}{dA} = \frac{1}{1-a(1-t)}$ (ii) $D = \overline{G} - t\left(\frac{\bar{I} + \overline{G}}{1-a(1-t)}\right)$ (iii) $\frac{dY}{dA} = \frac{1}{0.36} \cong 2.78$, $D = 1{,}000$

3.33 (i) $\frac{dC}{dq} = a(\alpha+1)q^{\alpha}$ (ii) $q = \left(\frac{F}{a\alpha}\right)^{\frac{1}{\alpha+1}}$

3.35 Slope of marginal revenue function: is -2β, slope of the average revenue function is $-\beta$

3.36 (i) $y = 162 - 2x^2$ (ii)



(iii) 162 (iv) 9 (v) $x = 6$, $y = 90$

3.37 (i) 100 (ii) *a*. year 12 *b*. year 9 (iii) 5.878 years

3.38 If $f'(p) < g'(p)$ the market is *stable*. This means that if disequilibrium prevails there will be a tendency for market price and quantity to move towards equilibrium.

If $g'(p) < f'(p)$ the market is *unstable*. If disequilibrium prevails the market price and quantity traded will move away from the equilibrium values.

3.39 (i) 0, $v'(x).\frac{x}{v(x)}$, $v'(x).\frac{x}{v(x)+F}$ (ii) -1, $\frac{xv'(x)}{v(x)} - 1$, $\frac{xv'(x)}{v(x)+F} - 1$

(iii) The elasticity of an average cost function is equal to the elasticity of its respective total cost function less one.

3.40 Marginal revenue is currently $4e^{-0.8} \cong 1.8$.

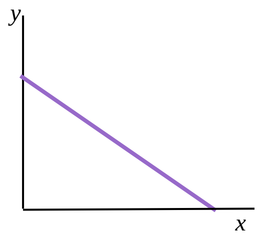
3.41 (i) $q = 48$, $p = 4$ (ii) $-\frac{25}{24}$, $\frac{1}{2}$ (iii) $q = 11 + \sqrt{1369 - 1200t}$ (iv) 12%

3.42 (i) $q_d = 52 - \frac{1}{3}p$, $q_s = -24 + 6p$ (ii) $p = \frac{228}{19+18s}$, $q = \frac{912+936s}{19+18s}$

(iii) $\varepsilon = \frac{1{,}368(1+s)}{912+936s}$ The elasticity is declines as the subsidy increases.

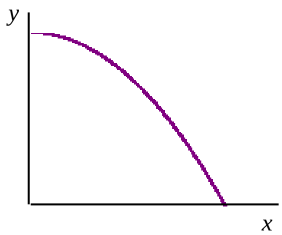
3.43

Country 1



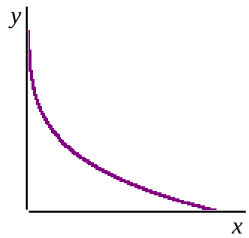
Opportunity cost is constant

Country 2



Opportunity cost is increasing

Country 3



Opportunity cost is decreasing

3.44 (i) $\dfrac{x^{-\delta}}{(x^{-\delta}+1)}$ (ii) $\dfrac{p^{-\delta}}{(p^{-\delta}-3)}$ Demand is elastic for all values of *p*. In (i) it is inelastic for all values of *p*.

3.45 (i) $p = 24,\ q = 180$

(ii) $p = -13 + \sqrt{1{,}369 + 20T}$, $q = -190 - 10T + 10\sqrt{1{,}369 + 20T}$ The equilibrium price increases and the equilibrium quantity falls as the tax increases.

(iii) $\frac{1}{4}$ of the tax is paid by consumers, $\frac{3}{4}$ of the tax is paid by producers

3.46 (i) $p = 10 + 10q_s^{\frac{1}{2}}$ (ii) $q = 4$, $p = 30$ (iii) -15, 3

(iv) $\varepsilon = \dfrac{2}{q_s^{\frac{1}{2}}} + 2$ Since $\dfrac{2}{q_s^{\frac{1}{2}}} > 0$, for finite values of q_s $\varepsilon > 2$. (v) $p = 20$, $\eta = -\dfrac{5}{3}$

3.48 (i) $p = 96$, $q = 8$ (ii) $\dfrac{dq_d}{dp} \cdot \dfrac{p}{q_d} = -2$, $\dfrac{dq_s}{dp} \cdot \dfrac{p}{q_s} = 1.5$ (iii) $q_d = \dfrac{768}{p - 64}$ (iv) $S = 64$

3.49 (i) $p = \dfrac{\alpha - \gamma}{\delta - \beta}$, $q = \dfrac{\alpha\delta - \gamma\beta}{\delta - \beta}$

(ii) $p = m(T) = \dfrac{\alpha - \gamma}{\delta - \beta} + \dfrac{\delta T}{\delta - \beta}$, $q = n(T) = \dfrac{\alpha\delta - \gamma\beta}{\delta - \beta} + \dfrac{\beta\delta T}{\delta - \beta}$

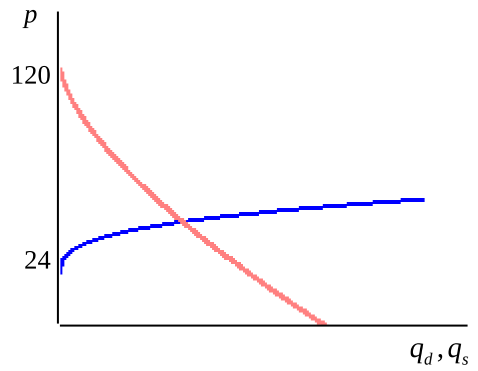
(iii) $\dfrac{dp}{dT} = \dfrac{\delta}{\delta - \beta}$ (iv) *a*. stable *b*. $t > \dfrac{2}{3}$

3.50 (i) 3 units of *X*, 48 units of *Y* (ii) 5 units of *X*, 132 units of *Y*, $y = 300 - 18x$

3.52 (i) $b\dfrac{p}{a + bp}$ (ii) Elastic when $a < 0$, inelastic when $a > 0$, unit elasticity when $a = 0$

3.53 (ii) $\eta = \dfrac{-p}{p + T}$ *a*. Increase price *b*. It tends to zero.

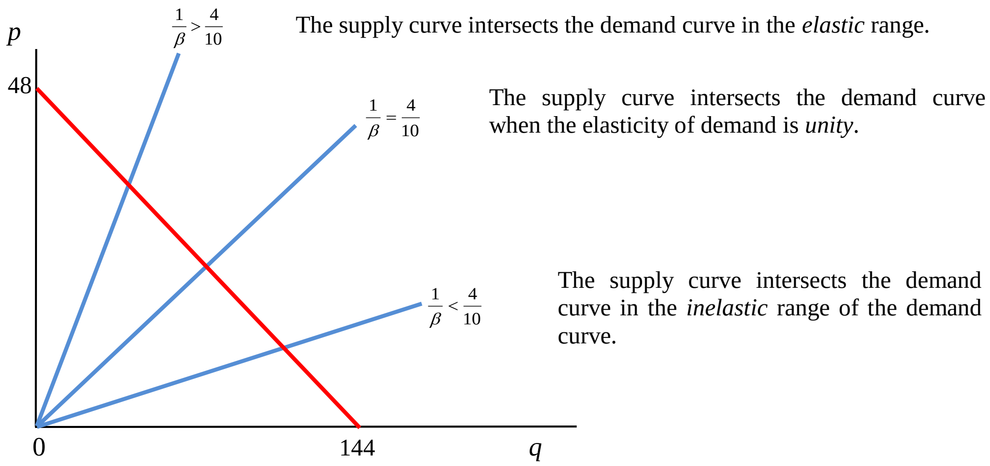
3.54 (i)



(ii) $q=27$, $p=48$ (iii) -1, 6 (iv) $p=20q_s^{\frac{1}{3}}+48$

3.55 (i) $p=\dfrac{132}{\beta+3}$, $q=\dfrac{144\beta+36}{\beta+3}$ (ii) $\dfrac{-11}{(1+4\beta)}$, elastic if $\beta<2.5$, inelastic if $\beta>2.5$

(iii)



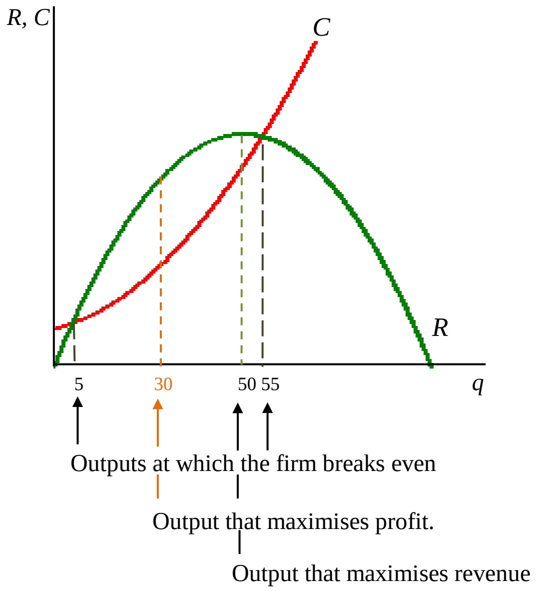
(iv) $\dfrac{22}{63}$

(v) £594, Consumers pay £3 less per unit after the introduction of the subsidy. Producers receive the remainder of the subsidy, £$6\frac{3}{7}$ per unit.

Chapter 4
Optimising a Function of One Variable

4.1 (i) $q = 30$, $p = 14$ (ii) Marginal revenue = marginal cost = 8

(iii)



4.2 $q = 124$, Marginal revenue = Marginal cost = 272

4.3 $E = 16{,}200p - 150p^2$, $p = 54$

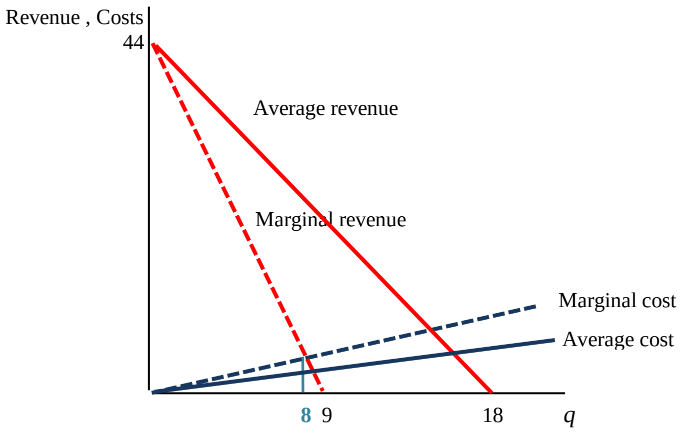
4.4 The firm will stop operating if price falls below 55.

4.5 (i) 12 (ii) 8

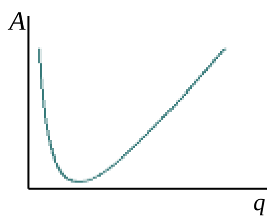
4.6 120

4.7 (i) yes (ii) $q = 18$, $\pi = 77$

4.8 $q = 8$, $\pi = 576$, $R = 640$, $C = 64$



4.9 (i) $C = q^2 - 6q + 16$ (ii) $q = 4$ (iii)



4.10 (i) 50 (ii) No

4.11 (i) $l = 60$, $q = 3{,}600$, $\pi = 22$ (ii) Output will not change. π will fall by 50.

4.12 $q = 12$, $C_m = 16$

4.13 (i) 25 (ii) Revenue maximisation (iii) $q = 68 - 0.2p$

4.14 (i) 5 (ii) $q = 6\frac{2}{3}$ (iii) Price falls by 2.5

4.15 (i) $q = 5$, $p = 2.5$ (ii) $\eta = -1$ (iii) $p = 3$ (iv) $q = 8$

4.16 (i) $\pi = 12q - 2q^2 - 16$

(ii) *a.* $q = 3 - 0.25T$ *b.* $\dfrac{dq}{dT} = -0.25$ *c.* $R_G = 3T - 0.25T^2$ *d.* $T = 6$ *e.* $q = 1.5$

4.17 65

4.18 $\dfrac{c(x)}{x} = c'(x)$

4.19 (i) a (ii) $\dfrac{C}{q} = \dfrac{ae^{bq}}{q}$, $\dfrac{dC}{dq} = abe^{bq}$ (iii) $q = \dfrac{1}{b}$, $\dfrac{C}{q} = \dfrac{dC}{dq} = aeb$

4.20 (i) $q = 20$, $\pi = 400$

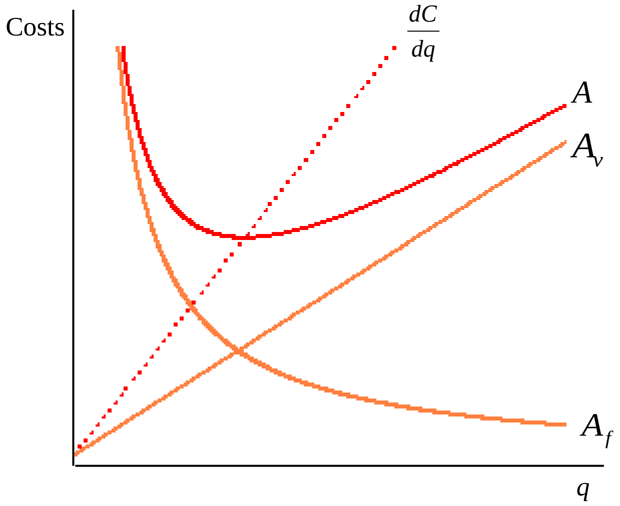
(ii) $p < 64$. At any price below 64 the cost of the output would be greater than the revenue the firm would receive from selling it.

4.21 (i) 252 (ii) $q = 2{,}331 - 1.25p$ (iii) $p = 252$

4.22 (i) 15 (ii) $p = 170$, $\pi = 400$

4.23 (i) $p = 122 - \frac{1}{4}q$

(ii) *a.*



b. $q = 120$, $\pi = 4{,}496$

4.24 (i) $p = 90$, $q = 15$ (ii) $T = 40$

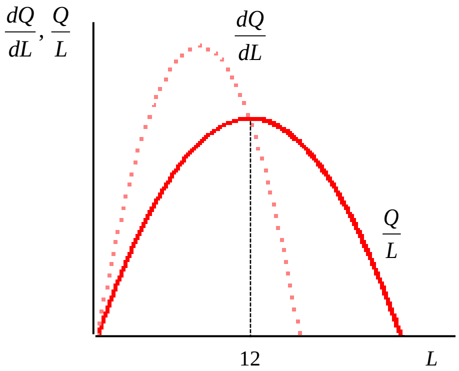
(iii) *a*. Consumers pay 37.5%, Producers pay 62.5%, $R_g = £192$

b. Consumers pay 37.5%, Producers pay 62.5%, $R_g = £192$

4.25 (i) 16 (ii) -2.75 (iii) Output does not change, profit falls by 48.

4.26 (i) Marginal product is increasing if $L < 8$ and diminishing if $L > 8$. (ii) $L = 12$

(iii)

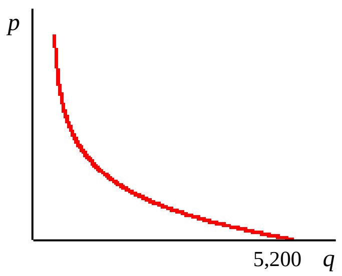


The average product of labour is maximised.

(iv) $L = 15$

4.27 (i) $L = 14$, $\pi = 338$ (ii) $w = 11$ (iii) £306

4.28 (i) (ii) 25



4.29 (i) $p = 1{,}200$, $q = 76{,}000$.

(ii) $p = 1{,}200 + 0.8T$, $q = 76{,}000 - 16T$, Price producer receives: $1{,}200 - 0.2T$

(iii) $T = 2{,}375$ (iv) $R_g = 90{,}250{,}000$, $q = 38{,}000$, $p = 3{,}100$, Producers receive 725

4.30 (i) $q = 100$, $\pi = 1{,}155$ (ii) $q = 65$

4.31 (i) $a = 172$, $b = 1$ (ii) $q = 1$ or $q = 9$

4.32 $L = 10$, $C = 300$.

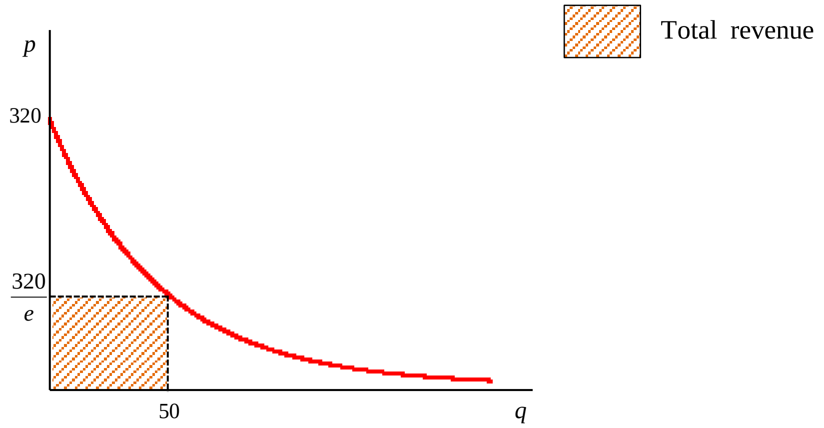
4.34 (i) $q = 50$ (ii) *a*. $q = 48$, $p = 308$ *b*. $-£784$ *c*. 25% *d*. £768

4.35 $\beta > 0$

4.36 $p = 5$, $q = 4.5$

4.37 $q = \dfrac{\varepsilon - \alpha}{2(\beta - \gamma)}$

4.38 (i) $q = 50$



(ii)

(iii) $\dfrac{16{,}000}{e}$

4.39 (i) 130 (ii) $\pi = 81{,}880$, $p = 988$ (iii) $q = 131.7$, $p = 987.7$

(iv) Consumer surplus is higher by 41.9 under perfect competition.

4.40 (i) $C = x^2 + 10x + 36$ (ii) $x = 6$: $\pi = 45$

(iii) $\dfrac{C}{x} = 22 = 22$, $\dfrac{dC}{dx} = 22$ The monopolist is operating at the minimum point of the average total cost curve.

(iv) $V = x^2 + 19x$

4.41 (i) $q = 34 - \frac{1}{9}S$, $p = 211 + \frac{7}{18}S$ (ii) $\dfrac{dq}{dS} = -\frac{1}{9}$, $\dfrac{dp}{dS} = \frac{7}{18}$

4.42 $x = \dfrac{\alpha - \varepsilon - T}{2(\beta + \delta)}$.

4.43 (i) $q = 20$, Yes (ii) $q = 26$, 20.8

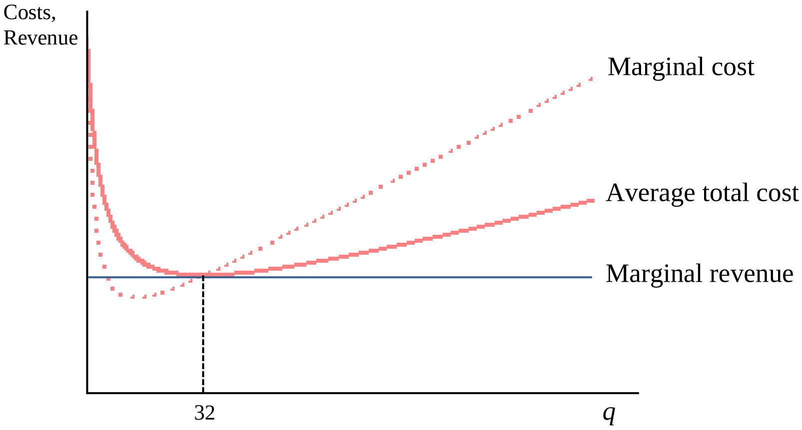
4.44 $q = \left[\dfrac{p}{\alpha\beta}\right]^{\frac{1}{\beta-1}}$, $\beta > 1$

4.45 (i) $\pi = 289{,}000$ (ii) $12 \le q \le 352$ (iii) $q = 352$

4.46 (i) $q = 32$, $p = 14.25$, $R = 456$

(ii) $\dfrac{dC}{dq} = 10 + \dfrac{324}{(q+4)^2} + 0.125q$,decreasing if $0 \le q < 13.3$ and increasing if $q > 13.3$

(iii)



4.47 (i) $p = 60$, $q = 6$ (ii) $p = 64$, $q = 4.4$

(iii) Consumers pay $\frac{1}{3}$ of the tax, Producers pay $\frac{2}{3}$ of the tax (iv) $\eta = -4$, $\varepsilon = 2$

(v) 22.5

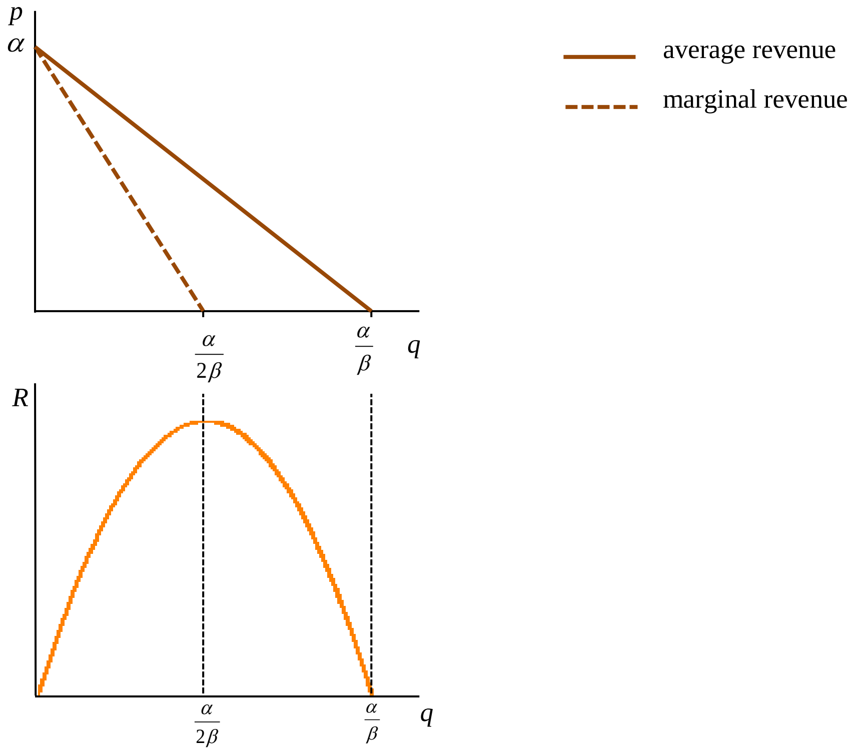
4.49 (i) $\eta = -bp$ (ii) $R = \frac{q}{b}\ln\left(\frac{a}{q}\right)$, $\frac{R}{q} = \frac{1}{b}\ln\left(\frac{a}{q}\right)$, $\frac{dR}{dq} = -\frac{1}{b} + \frac{1}{b}\ln\left(\frac{a}{q}\right)$ (iii) $q = \frac{a}{e}$

4.50 (i) $q = 54$ (ii) $q = 57$ (iii) $q = 54 + \frac{1}{6}S$, $S = 18$, $p = 114$

4.51 (i) $q = \frac{\alpha - a}{2\beta}$, $p = \frac{\alpha + a}{2}$, The average revenue curve must slope downwards.

(ii) Fixed costs have no effect on the profit-maximising output.

(iii)



$q = \frac{\alpha}{2\beta}$

4.52 The law of diminishing marginal utility must be operating.

4.53 (i) $q = 2{,}490$ (ii) The firm should pass on 50% of the increase in cost. (iii) 42%

4.55 (i) $p = \dfrac{\alpha + \gamma}{2}$ (ii) 50%

4.57 (i) $p = 160$, $q = 24$

(ii) *a*. Net revenue to the government: -129 *b*. Net revenue to the government: 223.8

c. Net revenue to the government: 291

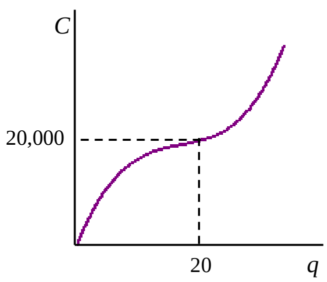
4.58 (i) $q = \sqrt{\dfrac{\delta}{\alpha}}$, Average cost $= 2\sqrt{\alpha\delta} + \beta$ (ii) $\pi = \dfrac{(\varepsilon - \phi\beta)^2}{4(1+\alpha\phi)} - \delta$, $p = \dfrac{\varepsilon + 2\alpha\phi\varepsilon + \phi\beta}{2\phi(1+\alpha\phi)}$

4.59 (i) $q = 11$, $R = 572$ (ii) $\dfrac{dC}{dq} = 43.04$ (iii) $q = 3{,}432 - 11p$

4.61 (i) $q = 20$, $p = 60$ (ii) -1 (iii) $q = 16$ (iv) $b = 0.01$

4.62 $t = \dfrac{\beta - b}{2}$

4.64 (i)



(ii) Marginal costs are decreasing for $0 < q < 20$ and increasing for $q > 20$.

(iii) 800 (iv) $q = \dfrac{400{,}000}{p}$, $R = 400{,}000$

4.65 (i) $q = 125$ (ii) -1 (iii) $\alpha = 0.005$

4.66 5

4.67 9

Chapter 5

Partial Differentiation

5.1 $\dfrac{\partial C}{\partial q_1} = 6q_1^2 + 4.8q_1q_2$, $\dfrac{\partial C}{\partial q_2} = 2q_2^{-0.5} + 2.4q_1^2$

5.2 (i) $\dfrac{\partial q}{\partial L} = 6L^{-\frac{1}{2}}K^{\frac{1}{4}} = \dfrac{6K^{\frac{1}{4}}}{L^{\frac{1}{2}}}$, $\dfrac{\partial q}{\partial K} = 3L^{\frac{1}{2}}K^{-\frac{3}{4}} = \dfrac{3L^{\frac{1}{2}}}{K^{\frac{3}{4}}}$ (ii) Yes

5.3 (i) $\dfrac{\partial U}{\partial q_1} = \dfrac{3}{4q_1}$, $\dfrac{\partial U}{\partial q_2} = \dfrac{1}{3q_2}$ (ii) Yes (iii) $f_{q_1q_1} < 0$ and $f_{q_2q_2} < 0$

5.4 (i) $\dfrac{\partial C}{\partial q_1} = 1.2q_1^2 - 12q_1 + 32 + 4q_2$, $\dfrac{\partial C}{\partial q_2} = 1.08q_2^2 - 8.64q_2 + 20 + 4q_1$

(ii) For good 1 marginal cost is decreasing for $0 \le q_1 < 5$ and increasing for $q_1 > 5$.

For good 2 marginal cost is decreasing for $0 \le q_2 < 4$ and increasing for $q_2 > 4$.

5.5 (i) $\dfrac{\partial C}{\partial q} = \frac{40}{3} w^{\frac{1}{4}} r^{\frac{1}{2}} q^{\frac{1}{3}}$ (iii) $\dfrac{\partial C}{\partial w} \cdot \dfrac{w}{C} = \dfrac{1}{4}$

5.6 $\dfrac{\partial q_a^d}{\partial p_a} \cdot \dfrac{p_a}{q_a^d} = -\alpha_1 \left(\dfrac{p_a}{\alpha_0 - \alpha_1 p_a + \alpha_2 p_b + \alpha_3 Y} \right)$, $\dfrac{\partial q_a^d}{\partial p_b} \cdot \dfrac{p_b}{q_a^d} = \alpha_2 \left(\dfrac{p_b}{\alpha_0 - \alpha_1 p_a + \alpha_2 p_b + \alpha_3 Y} \right)$

$\dfrac{\partial q_a^d}{\partial Y} \cdot \dfrac{Y}{q_a^d} = \alpha_3 \left(\dfrac{Y}{\alpha_0 - \alpha_1 p_a + \alpha_2 p_b + \alpha_3 Y} \right)$

Good *A* is a normal good. Goods *A* and *B* are substitutes

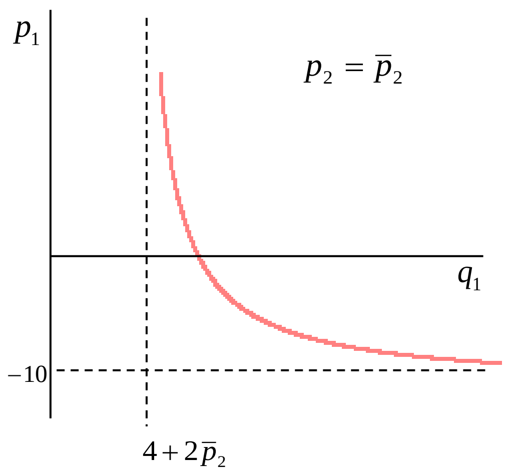
5.7 (i) $\dfrac{\partial q_x}{\partial p_x} \cdot \dfrac{p_x}{q_x} = \dfrac{-\beta_1 p_y p_x}{\alpha_1 p_y^2 - \beta_1 p_x p_y + \gamma_1 Y}$, $\dfrac{\partial q_x}{\partial p_y} \cdot \dfrac{p_y}{q_x} = \dfrac{2\alpha_1 p_y^2 - \beta_1 p_x p_y}{\alpha_1 p_y^2 - \beta_1 p_x p_y + \gamma_1 Y}$

$\dfrac{\partial q_x}{\partial Y} \cdot \dfrac{Y}{q_x} = \dfrac{\gamma_1 Y}{\alpha_1 p_y^2 - \beta_1 p_x p_y + \gamma_1 Y}$

(ii) $\dfrac{\beta_1}{2\alpha_1} < \dfrac{p_y}{p_x} < \dfrac{2\alpha_2}{\beta_2}$

5.8 (i) $q_1 = 14$, $q_2 = 16$ (ii) Yes (iii) These goods are substitutes.

(iv)



5.9 (i) $\frac{\partial Q}{\partial L} = \frac{1}{2}(L-3)^{-\frac{1}{2}}(K-2)^{\frac{3}{4}}$, $\frac{\partial Q}{\partial K} = \frac{3}{4}(L-3)^{\frac{1}{2}}(K-2)^{-\frac{1}{4}}$ (ii) Yes

(iii) The marginal product of labour increases as K increases.

5.10 (i) $\frac{\partial Q}{\partial L} = \frac{1}{16} + \frac{3}{16}K^{\frac{1}{2}}L^{-\frac{1}{2}}$, $\frac{\partial Q}{\partial K} = \frac{9}{16} + \frac{3}{16}L^{\frac{1}{2}}K^{-\frac{1}{2}}$ (ii) Yes

5.11 (i) Constant returns to scale. The firm's long-run average cost curve will be a horizontal straight line.

(ii) $\frac{\partial q}{\partial L} = A\alpha L^{\alpha-1}K^{1-\alpha} + B$, $\frac{\partial q}{\partial K} = A(1-\alpha)L^{\alpha}K^{-\alpha} + C$, $0 < \alpha < 1$.

(iii) $\frac{A\alpha L^{\alpha}K^{1-\alpha} + BL}{A\alpha L^{\alpha}K^{1-\alpha} + BL + CK}$. Total payments to factors will exhaust total product exactly.

(iv) $MRS_{K,L} = \frac{A(1-\alpha)L^{\alpha}K^{-\alpha} + C}{A\alpha L^{\alpha-1}K^{1-\alpha} + B}$

5.12 (i) Decreasing returns to scale. (ii) $\frac{\partial Q}{\partial L} = \frac{3K^{\frac{1}{4}}}{2L^{\frac{1}{2}}} > 0 \quad \forall\, K, L$, $\frac{\partial Q}{\partial K} = \frac{3L^{\frac{1}{2}}}{4K^{\frac{3}{4}}} > 0 \quad \forall\, K, L$

(iii) The exponents give the elasticities of output with respect to the factor inputs.

(iv) $K = 100$ and $L = 90$, $K = 9$ and $L = 300$.

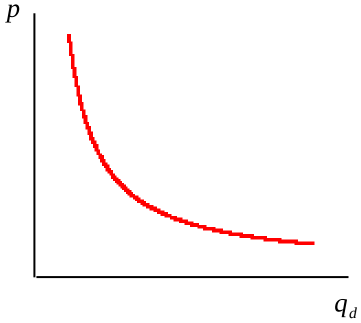
5.13 (i) $\frac{\partial q}{\partial p} \cdot \frac{p}{q} = -3$, $\frac{\partial q}{\partial Y} \cdot \frac{Y}{q} = \frac{1}{2}$

(ii) $\frac{\partial \ln q}{\partial \ln p} = -3$, $\frac{\partial \ln q}{\partial \ln Y} = \frac{1}{2}$

The derivative of the logarithm of q with respect to the logarithm of p gives the price elasticity and the derivative of the logarithm of q with respect to the logarithm of Y gives the income elasticity.

5.14 (i) 0 (ii) Yes (iii) Complements

5.15 (i)



Total revenue is constant and equal to 96.

(ii) $\frac{\partial q_d}{\partial p}\cdot\frac{p}{q_d}=-1$, $\frac{\partial q_d}{\partial p_1}\cdot\frac{p_1}{q_d}=0.5$, $\frac{\partial q_d}{\partial y}\cdot\frac{y}{q_d}=0.5$

The good has a constant elasticity of unity and it is a substitute for the good whose price is p_1. This good is a normal good that is income inelastic (a necessity).

(iii) 0

(iv) $\frac{\partial q_d}{\partial p}p+\frac{\partial q_d}{\partial p_1}p_1+\frac{\partial q_d}{\partial y}y=0q_d$, $\underbrace{\frac{\partial q_d}{\partial p}\cdot\frac{p}{q_d}+\frac{\partial q_d}{\partial p_1}\cdot\frac{p_1}{q_d}}_{\text{price elasticities}}=\underbrace{-\frac{\partial q_d}{\partial y}\cdot\frac{y}{q_d}}_{\text{negative of income elasticity}}$

5.16 (i) $p_1=8, q_1=27$, $p_2=11, q_2=36$ and $p_3=5, q_3=4$.

(ii) $\frac{\partial q_{1d}}{\partial p_1}\cdot\frac{p_1}{q_{1d}}=\frac{-2p_1}{45-2p_1+3p_2-7p_3}$, $\frac{\partial q_{1d}}{\partial p_2}\cdot\frac{p_2}{q_{1d}}=\frac{3p_2}{45-2p_1+3p_2-7p_3}$

$\frac{\partial q_{1d}}{\partial p_3}\cdot\frac{p_3}{q_{1d}}=\frac{-7p_3}{45-2p_1+3p_2-7p_3}$

(iii) $\frac{\partial q_{1d}}{\partial p_1}\cdot\frac{p_1}{q_{1d}}=\frac{16}{27}$, $\frac{\partial q_{1d}}{\partial p_2}\cdot\frac{p_2}{q_{1d}}=\frac{11}{9}$, $\frac{\partial q_{1d}}{\partial p_3}\cdot\frac{p_3}{q_{1d}}=-\frac{35}{11}$

5.17 (i) $Y_e=\frac{\alpha-\beta\gamma+I+G}{1-\beta+\beta\delta}$ (iii) $Y_e=1{,}000$

5.18 (i) *a*. $Y_e=\frac{a+\bar{I}+\bar{G}-bT}{1-b}$, $C_e=\frac{a+b(\bar{I}+\bar{G})-bT}{1-b}$

b. $Y_e=\frac{a+\bar{I}+\bar{G}}{1-b+bt}$, $C_e=\frac{a+b(1-t)(\bar{I}+\bar{G})}{1-b+bt}$, $T_e=\frac{t(a+\bar{I}+\bar{G})}{1-b+bt}$

(ii) With lump-sum taxation: $\Delta Y_e=-\frac{bT}{1-b}$

With proportional taxation: $\Delta Y_e=\frac{-bt(a+\bar{I}+\bar{G})}{(1-b+bt)(1-b)}$

Taxation reduces the equilibrium level of income.

With lump-sum taxation: $\frac{\partial Y}{\partial \bar{I}}=\frac{\partial Y}{\partial \bar{G}}=\frac{\partial Y}{\partial a}=\frac{1}{1-b}$

With proportional taxation: $\frac{\partial Y}{\partial \bar{I}}=\frac{\partial Y}{\partial \bar{G}}=\frac{\partial Y}{\partial a}=\frac{1}{1-b+bt}$

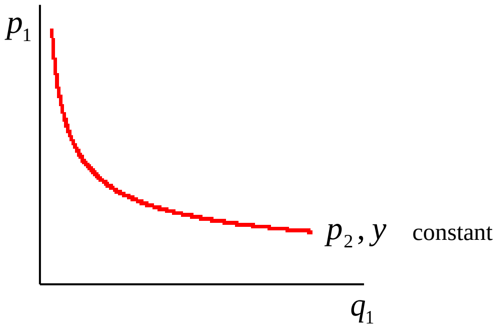
Taxation reduces the values of the multipliers associated with autonomous expenditure.

5.19 (i) Homogeneous of degree $\alpha+\beta+\delta$. (ii) The own-price elasticity of demand $=\alpha$, $\alpha<0$

(iii) The income elasticity of demand $=\delta$, $\delta>0$ (iv) $\beta<0$

(v) *a*. The sum of the price elasticities is equal to the negative of the income elasticity.

b.



c. Reduce price.

5.20 $x=10$ and $y=8$, $x=30$ and $y=4$ (ii) Yes

5.22 (i) $f(L,K)$ is homogeneous of degree $\alpha+\beta$, $g(L,K)$ is not homogeneous.

5.23 (i) $Y_e=\dfrac{a_0+I_0+G_0+X_0}{1-a_1+a_1t+m}$, $C_e=\dfrac{a_0(1+m)+a_1(1-t)(I_0+G_0+X_0)}{1-a_1+a_1t+m}$

$T_e=\dfrac{t(a_0+I_0+G_0+X_0)}{1-a_1+a_1t+m}$, $M_e=\dfrac{m(a_0+I_0+G_0+X_0)}{1-a_1+a_1t+m}$

(ii) $\dfrac{\partial Y_e}{\partial I_0}=\dfrac{1}{1-a_1+a_1t+m}$. $\dfrac{\partial Y_e}{\partial I_0}$ falls for an increase in the marginal propensity to import.

(iii) $\dfrac{\partial Y_e}{\partial m}=-\dfrac{(a_0+I_0+G_0+X_0)}{(1-a_1+a_1t+m)^2}$,

$\dfrac{\partial T_e}{\partial m}=-\dfrac{t(a_0+I_0+G_0+X_0)}{(1-a_1+a_1t+m)^2}$, $\dfrac{\partial M_e}{\partial m}=\dfrac{(1-a_1+a_1t)(a_0+I_0+G_0+X_0)}{(1-a_1+a_1t+m)^2}$

(iv) $\dfrac{\partial Y_e}{\partial I_0}=\dfrac{1}{1-a_1+a_1t-t+m}$, This multiplier is larger than the multiplier that results when a balanced budget is not operated.

5.24 (i) $\dfrac{C}{Y}=\dfrac{\alpha}{Y}+\beta$, $\dfrac{dC}{dY}=\beta$

(ii) $Y=\dfrac{\alpha+\gamma}{1-\beta}-\dfrac{\delta}{1-\beta}r$ and $Y=\dfrac{\overline{M}-d}{e}+\dfrac{f}{e}r$

$$Y_e=\frac{(\alpha+\gamma)+\left(\delta/f\right)(\overline{M}-d)}{(1-\beta)+\left(\delta e/f\right)},\quad r_e=\frac{(\alpha+\gamma)e-(1-\beta)(\overline{M}-d)}{f(1-\beta)+\delta e}$$

(iii) $\frac{\partial M_d}{\partial r}\cdot\frac{r}{M_d}=\frac{-fr}{d+eY-fr}$ (v) $\frac{\partial Y_e}{\partial \gamma}=\frac{1}{(1-\beta)+\left(\delta e/f\right)}$

5.25 The average product of labour is $\frac{Q}{L}=\frac{AL^{\alpha}K^{\beta}}{L}=AL^{\alpha-1}K^{\beta}$.

$\frac{\partial\left(\frac{Q}{L}\right)}{\partial L}=(\alpha-1)AL^{\alpha-2}K^{\beta}<0$ for $\alpha<1$ since A, L and K are positive

5.26 (i) Not homogeneous (iii) $\gamma<1$

5.27 (i) No (ii) Yes (iii) Labour: α, Capital: β, $\alpha=\beta$ (v) $\sigma=1$

5.28 (i) $L>\frac{A}{3BK}$ (iii) $-\frac{dL}{dK}=\frac{L}{K}$, Yes (iv) $\sigma=1$

5.29 (i) Constant (ii) $\frac{\partial Q}{\partial L}=L^{-(\alpha+1)}(L^{-\alpha}+K^{-\alpha})^{-\frac{1}{\alpha}-1}$, $\frac{\partial Q}{\partial K}=K^{-(\alpha+1)}(L^{-\alpha}+K^{-\alpha})^{-\frac{1}{\alpha}-1}$

(iii) $\frac{L^{-\alpha}}{(L^{-\alpha}+K^{-\alpha})}$ (iv) $MRTS_{K,L}=-\left(\frac{K}{L}\right)^{-\frac{1}{\alpha}-1}$, Yes

Chapter 6

Optimising a Function of Two Variables

6.1 $q_1=48$, $q_2=40$

6.2 (i) $q_1=18$ and $p_1=144$, $q_2=4$ and $p_2=120$ (ii) $q_1=90-\frac{2}{3}p_1$

6.3 (i) $q_1+q_2=240$, $p=26$, $\pi_1=1280$, $\pi_2=3200$

(ii) $q_1+q_2=200$, $p=30$, $\pi_1=960$, $\pi_2=3840$, Firm 1 must pay firm 2 at least 320.

6.4 (i) 34 units in submarket 1 and 6 units in submarket 2 (ii) $\eta_1\cong-1.647$, $\eta_2=-23$

6.5 (i) $q_1=11$, $q_2=2$, $\pi=367$ (ii) $p_1=48$, $p_2=28$

(iii) Marginal cost in market *i* = marginal revenue in market *i* = 26

6.6 (i) $q_1=36$, $q_2=196$ (ii) $q_1=7$, $q_2=22$, $\pi=-532$, The firm should shut down

6.7 (i) $x=10$, $y=40$, $\pi=1{,}620$ (ii) The firm will make a loss of 300.

6.8 (i) $q_a=50$, $q_b=30$, $\pi=12{,}500$ (ii) $p_a=210$, $p_b=280$

(iii)

Variable	Maximisation of: Profit	Maximisation of: Revenue	Consequences of the change in the firm's objective from profit maximisation to revenue maximisation
q_a	50	130	Increase of 80 in the output of good *A*
q_b	30	50	Increase of 20 in the output of good *B*
			Total increase in output is 100
π	12,500	–5,500	Profit has fallen by 18,000. The firm is making a *loss* of 5,500

6.9 (i) $q_1 = 21,\ q_2 = 45$ (ii) Marginal revenue in market *i* = marginal cost = 20

6.10 (i) $q_1 = 8,\ q_2 = 7,\ p_1 = 60,\ p_2 = 110,\ \pi = 650$

(ii) $\dfrac{dq_1}{dp_1} \cdot \dfrac{p_1}{q_1} = -3,\ \dfrac{dq_2}{dp_2} \cdot \dfrac{p_2}{q_2} = -\dfrac{11}{7}$, Yes because demand is relatively more elastic.

(iii) Profit will fall by 200.

6.11 (i) $q_1 = \dfrac{\alpha_1 - \gamma}{2\beta_1},\ q_2 = \dfrac{\alpha_2 - \gamma}{2\beta_2},\ \alpha_1 > \alpha_2$ (ii) $p_1 = \dfrac{\alpha_1 + \gamma}{2},\ p_2 = \dfrac{\alpha_2 + \gamma}{2},\ \alpha_1 > \gamma,\ \ \alpha_2 > \gamma$

6.12 (i) $m = 16,\ q = 11,\ \pi = 994$ (ii) Output should be reduced to 6. Profit will fall by 226.

6.14 (i) $q_1 = 50,\ q_2 = 15,\ \pi = 4{,}725$

(ii) $p_1 = 120,\ p_2 = 77.5,\ \dfrac{dq_1}{dp_1} \cdot \dfrac{p_1}{q_1} = -2.4,\ \dfrac{dq_2}{dp_2} \cdot \dfrac{p_2}{q_2} = -10\tfrac{1}{3}$

6.15 (i) $q_1 = 4,\ q_2 = 5,\ p_1 = 50,\ \ p_2 = 35$

(iii) $p = 40,\ \ \pi = 200$, 15 less than that obtainable with price discrimination

6.16 $L = 4$, $K = 3.75$, $\pi = 176.5$

6.17 (i) $q_1 = 8,\ q_2 = 16,\ \pi = 753$

(ii) *a*. $q = 15,\ q_1 = 5,\ q_2 = 10,\ p = 70,\ \pi = 735$

b. Output and price unchanged, profit falls by 100.

c. $p = 76\ q = 12,\ q_1 = 4,\ q_2 = 8,\ ,\ \pi = 535$, the sales tax

6.18 (i) $x = 5,\ y = 3$ (ii) $\dfrac{dx}{dp_x} \cdot \dfrac{p_x}{x} = -4\tfrac{1}{5},\ \dfrac{dy}{dp_y} \cdot \dfrac{p_y}{y} = -2\tfrac{1}{3}$

(iii) $x = 5 - \tfrac{5}{18}t,\ y = 3 + \tfrac{1}{18}t,\ t = 18,\ y = 4,\ t = 9$

6.19 (i) $q_1 = 6$, $q_2 = 6$, $p_1 = p_2 = 5\tfrac{1}{2}$ (ii) $q_1 = 7\tfrac{1}{5}$, $q_2 = 7\tfrac{1}{5}$, $p_1 = p_2 = 4\tfrac{3}{5}$

(iii) $q_1 = 7\tfrac{1}{14}$, $q_2 = 7\tfrac{5}{7}$, $p_1 = 4\tfrac{15}{28}$, $p_2 = 4\tfrac{3}{8}$

6.20 (i) No (ii) $\dfrac{\partial Q}{\partial L} = \tfrac{1}{2}L^{-\frac{1}{2}}$, $\dfrac{\partial Q}{\partial K} = \tfrac{2}{3}K^{-\frac{1}{3}}$, Both marginal products are diminishing.

(iii) $L = 9$, $K = 8$, $Q = 7$, $\pi = 34$

6.21 *a.* $p = 3$, $q_1 = \tfrac{1}{9}$, $q_2 = \tfrac{1}{9}$, $\pi = \tfrac{4}{15}$

b. $p_1 = 3.6$, $p_2 = 2.7$, $q_1 = \tfrac{25}{324}$, $q_2 = \tfrac{1000}{6561}$,

6.22 (i) Returns to scale are decreasing. (iii) $L = 16p^2$, $K = 4p^2$ (iv) $p = 4$, $L = 256$, $K = 64$

6.23 (i) $\dfrac{\partial Q}{\partial L} = \dfrac{1}{1+L}$, $\dfrac{\partial Q}{\partial K} = \dfrac{1}{1+K}$ (ii) Yes

(iii) $L = \dfrac{p}{w} - 1$, $K = \dfrac{p}{r} - 1$, $Q = \ln(1 + \dfrac{p}{w} - 1) + \ln(1 + \dfrac{p}{r} - 1) = \ln\left(\dfrac{p}{w}\right) + \ln\left(\dfrac{p}{r}\right) = \ln\left(\dfrac{p^2}{rw}\right)$

6.24 $q = 24$, $a = 1{,}296$.

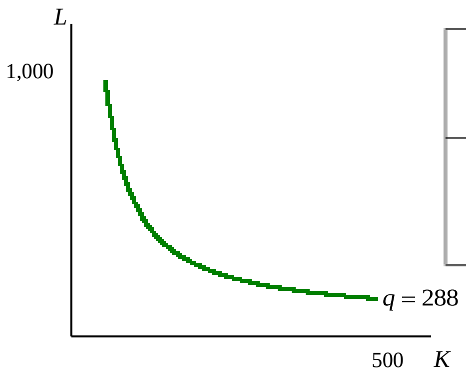
6.25 (i) $q_1 = 18$, $q_2 = 12$ (iii) $\eta = -1.0417$ (iv) No

6.26 (ii) $\dfrac{\partial^2 \pi}{\partial x^2} < 0$, $\dfrac{\partial^2 \pi}{\partial y^2} < 0$, $\left(\dfrac{\partial^2 \pi}{\partial x^2}\right)\left(\dfrac{\partial^2 \pi}{\partial y^2}\right) - \left(\dfrac{\partial^2 \pi}{\partial x \partial y}\right)^2 > 0$ (iii) $\pi = 375$

6.27 (ii) $q_1 = \dfrac{\alpha}{3\beta}$, $q_2 = \dfrac{\alpha}{3\beta}$, $p = \dfrac{\alpha}{3}$

6.28 (i) $L = 324$, $K = 144$, $\pi = 1{,}296$

(ii)



Inputs		Cost
L	K	$C = 4L + 9K$
46,656	1	186,633
324	144	2,592
64	729	6,817

6.29 $q_1 = \dfrac{(\alpha - \gamma)\delta - \beta\gamma}{2\beta\delta}$, $q_2 = \dfrac{\gamma}{2\delta}$

6.30 (i) $MRS_{L,K} = \dfrac{K}{2L}$, The marginal rate of substitution of labour for capital is diminishing.

(ii) $L = 400$, $K = 320$

6.31 (i) $K = 64$, $L = 4$ (ii) $p = 0.05$, $\pi = 25.6$ (iii) $K = 2^{\frac{19}{3}}$, $Q = 2^{\frac{32}{3}}$, $p = \dfrac{1.6}{2^{\frac{16}{3}}}$

6.33 (ii) $L = 25$, $K = 16$, $\pi = 123$

(iii) $L = \left(\dfrac{\alpha}{w}\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(\dfrac{\beta}{r}\right)^{\frac{\beta}{1-\alpha-\beta}} (pA)^{\frac{1}{1-\alpha-\beta}}$, $K = \left(\dfrac{\beta}{r}\right)^{\frac{1-\alpha}{1-\alpha-\beta}} \left(\dfrac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha-\beta}} (pA)^{\frac{1}{1-\alpha-\beta}}$

The profit function is homogeneous of degree one so if there is a combination of K and L for which profit is positive it will be always be possible for the firm to increase profit by increasing both inputs in the same proportion. The combination of inputs it should use to maximise profit is therefore indeterminate.

Chapter 7

Constrained Optimisation II

7.1 $x = 7$, $y = 14$

7.2 $x = 2$, $y = 5$, $\lambda = 1$

7.3 (i) $\dfrac{\partial U}{\partial x} = y$, $\dfrac{\partial U}{\partial y} = x$ (ii) $x = 2$, $y = 4$ (iii) $x = 3$, $y = 3$ The consumer is better off.

7.4 $L = 4$, $Q = 74$

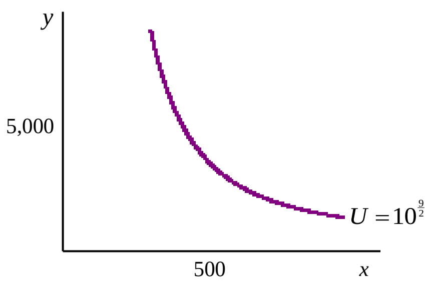
7.5 $q_1 = 200$, $q_2 = 100$, $q_3 = 700$

7.6 $q_1 = 40$, $q_2 = 60$, $C = 2{,}340$

7.7 (i) Yes (ii) $q_1 = 10$, $q_2 = 18$ The marginal utility of money = $\frac{2}{15}$.

7.8 $q_1 = 50$, $q_2 = 150$, $\dfrac{\partial C}{\partial q_1} = 4q_1 + q_2 = 350$, $\dfrac{\partial C}{\partial q_2} = q_1 + 2q_2 = 350$

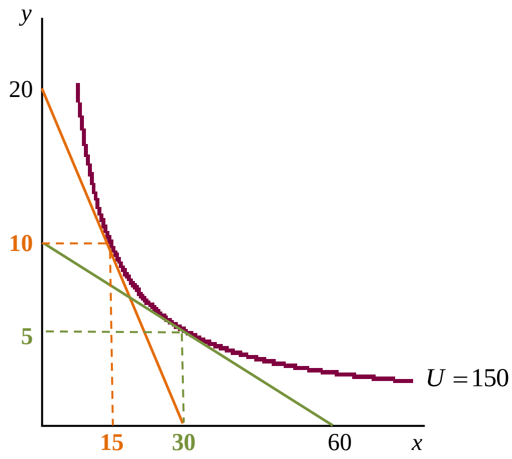
7.9 (i) $x = 1{,}000$, $y = 1{,}000$ (ii) $x = 500$ and $y = 4{,}000$, $x = 800$ and $y = 1{,}562.5$.



7.10 (i) Yes, If the output of both plants is doubled total cost will increase by 4 fold.

(ii) $q_1 = 48$, $q_2 = 32$, $C = 25{,}600$

7.11 (i) $x = 15$, $y = 10$ (ii) 120 (iii)



7.12 (i) Increasing (ii) $\frac{\partial Q}{\partial L} = 3K$, $\frac{\partial Q}{\partial K} = 3L$ (iii) $L = 4$, $K = 2$, $Q = 24$ (iv) $\frac{\partial Q}{\partial L} = 6$, $\frac{\partial Q}{\partial K} = 12$

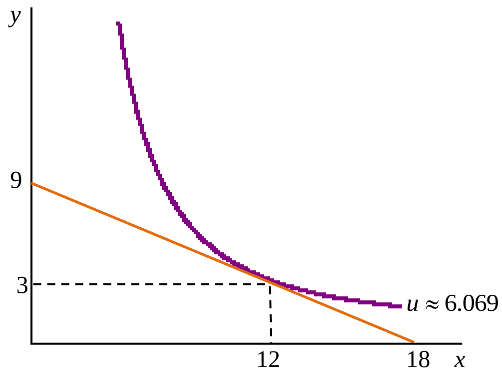
7.13 (i) $L = 225$, $K = 625$ (ii) $Q = 630$, $\pi = 2835$

7.14 (i) $\frac{\partial Q}{\partial L} = \frac{2}{9}L^{-\frac{2}{3}}$, $\frac{\partial Q}{\partial K} = \frac{1}{3}K^{-\frac{2}{3}}$, Yes (ii) $L = 540$, $K = 160$

7.15 (i) $U = 700$ (ii) Utility will fall by 140.

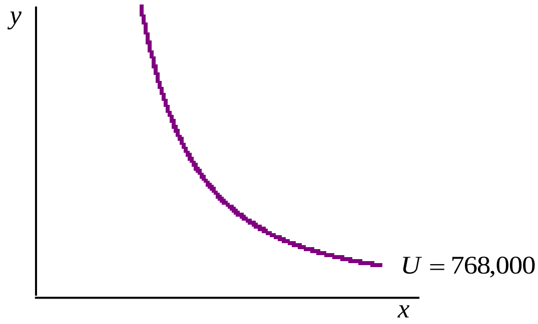
7.16 (i) $\frac{\partial u}{\partial x} = \frac{2}{x}$, $\frac{\partial u}{\partial y} = \frac{1}{y}$ (ii) $x = 12$, $y = 3$, $u = \ln 432$

(iii)



7.17 (i) $K = 3\overline{Q}^{\frac{1}{2}}$, $L = 12\overline{Q}^{\frac{1}{2}}$ (ii) $C = 96\overline{Q}^{\frac{1}{2}}$ (iii) $\frac{dC}{dQ} = \frac{48}{\overline{Q}^{\frac{1}{2}}}$

7.18 (i) $x = 80$, $y = 60$ (ii) $U = 768{,}000$ (iii) $\frac{\partial U}{\partial x} = 19{,}200$, $\frac{\partial U}{\partial y} = 12{,}800$



7.19 $x = 18$, $y = 8$

7.20 (i) $\frac{\partial U}{\partial x} = 10 - 2x$, $\frac{\partial U}{\partial y} = 30 - 6y$, $x > 5$, $y > 5$ (ii) $x = 3.5$, $y = 4$, $U = 94.75$

7.21 (i) $x = 100$, $y = 16$, $U = 14$ (iii) The marginal utility of money = 0.0625.

7.22 (i) $x = 8$, $y = 20$ (ii) $y = \frac{40}{p_y}$.

7.23 (i) $q_d = 15$, $q_f = 13$ $p_d = 50$, $p_f = 72$ (ii) $q_d = 9$, $q_f = 10$ Profit falls by 108.

7.24 (ii) $x = 64$, $y = 96$, $S = 417{,}792$ (iii) $-\frac{dy}{dx} = \frac{4y}{3x}$, Yes

7.25 (i) $x_1 = \frac{3y}{5p_1}$, $x_2 = \frac{2y}{5p_2}$ (ii) $x_1 = 36$, $x_2 = 18$ (iii) No

7.26 (i) $y = \frac{1{,}600 + 236p_y}{32 + p_y^2}$ (ii) $y = 30$, $x = 34$, $U = 6{,}588$ (iii) $\frac{\partial U}{\partial x} = 32$, $\frac{\partial U}{\partial y} = 80$, 8

7..27 (i) $L = \frac{Q^*}{4}$, $K = \frac{Q^*}{16}$ (ii) $C = 12.5Q^*$, $\frac{dC}{dQ^*} = 12.5$ (iii) $L = 5$, $K = 1.25$

7.28 $L = 3{,}200$, $K = 375$

7.29 (i) $\alpha = 0.2$ (ii) The marginal utility of money ≅ 0.35.

Chapter 8

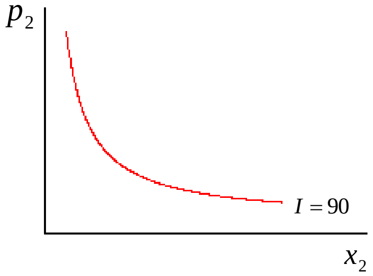
Constrained Optimisation II

8.2 (i) $q_1 = 36$, $q_2 = 15$, $\pi = 321.5$ (ii) $q_1 = 34$, $q_2 = 17$ Profit falls by 3.

8.3 (i) $x = \frac{m}{p_x} - 1$, $y = \frac{p_x}{p_y}$ (ii) 0 (iii) No (iv) $\frac{\partial x}{\partial p_x} \cdot \frac{p_x}{x} = -\frac{m}{m - p_x}$, $\frac{\partial x}{\partial m} \cdot \frac{m}{x} = \frac{m}{m - p_x}$

8.4 (ii) $x_1 = \dfrac{I}{3p_1}$, $x_2 = \dfrac{2I}{3p_2}$ (iii) $\dfrac{\partial x_2}{\partial p_2} \cdot \dfrac{p_2}{x_2} = -1$, $\dfrac{\partial x_2}{\partial I} \cdot \dfrac{I}{x_2} = 1$

(iv)



(v) The same demand functions as those obtained in (ii).

8.5 (i) $\dfrac{\partial u}{\partial x} = \dfrac{\alpha}{x}$, $\dfrac{\partial u}{\partial y} = \dfrac{\beta}{y}$, $\alpha > 0$, $\beta > 0$ (ii) $x = \dfrac{\alpha I}{(\alpha + \beta) p_x}$, $y = \dfrac{\beta I}{(\alpha + \beta) p_y}$.

(iii) $\lambda = \dfrac{\alpha + \beta}{I}$ so it depends on the consumer's income and the parameters of the utility function. It will be lower at *B* than at *A*.

8.6 (i) Yes

(ii) $x = \dfrac{72p_y^2 - 25p_x p_y + Ip_x}{p_x^2 + 2p_y^2}$, $y = \dfrac{25p_x^2 - 72p_x p_y + 2Ip_y}{p_x^2 + 2p_y^2}$, $\lambda = \dfrac{36p_x + 25p_y - I}{p_x^2 + 2p_y^2}$

(iii) No

8.7 (i) $q = 248{,}832$ (ii) Output will increase by 2 units.

8.8 (i) $2{,}880$, £67.2 (ii) $u = 3{,}174$

8.9 (i) $\alpha = 0.25$ (ii) $U = 50$ (iii) $\lambda = 0.1953125$

8.10 (i) $L = 0.15625\overline{Q}^2$, $K = 0.025\overline{Q}^2$, $c(Q) = 0.75\overline{Q}^2$ (ii) $p = 90$

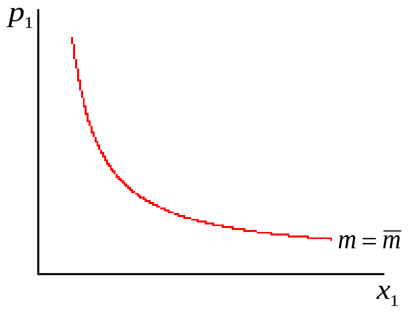
8.11 (ii) $x_1 = \dfrac{M}{3p_1}$, $x_2 = \dfrac{M}{3p_2}$, $x_3 = \dfrac{M}{3p_3}$ (iii) $u = \ln \dfrac{M^3}{27 p_1 p_2 p_3}$, $\lambda = \dfrac{3}{M}$

8.12 (i) $x = 42\left(\dfrac{w}{p}\right) + 0.25\left(\dfrac{m}{p}\right)$ (ii) $U = \dfrac{1}{p^{0.25}}\left(\dfrac{3}{w}\right)^{0.75}(42w + 0.25m)$, $\dfrac{\partial U}{\partial x} = 0.25\left(\dfrac{3p}{w}\right)^{0.75}$

(iii) $L_s = 42 - 0.75\left(\dfrac{m}{w}\right)$ (iv) Utility will be unchanged.

8.13 (i) $\alpha > 0$, $\beta > 0$, $\gamma > 0$ (ii) $x_1 = \dfrac{\alpha m}{p_1}$, $x_2 = \dfrac{\beta m}{p_2}$, $x_3 = \dfrac{\gamma m}{p_3}$

(iii) No



(iv) Yes if $\beta > 0$

8.14 (i) $x = \frac{w}{p}168 + \frac{m}{p} - \frac{1}{32}\left(\frac{w}{p}\right)^{-\frac{1}{4}}$, $L_s = 168 - L = 168 - \frac{1}{32}\left(\frac{w}{p}\right)^{-\frac{5}{4}}$ (ii) L_s will increase.

8.15 (i) $x_1 = \frac{2(w24+m)}{15p_1}$, $x_2 = \frac{w24+m}{5p_2}$, $L = \frac{2(w24+m)}{3w}$ (ii) $L_s = 8 - \frac{2m}{3w}$ (iii) 8

(iv) The supply of labour will increase if there is a small increase in the wage rate.

8.16 (i) $L = \left(\frac{\overline{q}}{96}\right)^{\frac{4}{3}}\left(\frac{2r}{w}\right)^{\frac{1}{3}}$, $K = \left(\frac{\overline{q}}{96}\right)^{\frac{4}{3}}\left(\frac{w}{2r}\right)^{\frac{2}{3}}$.

(ii) $C = \frac{3}{2}\left(\frac{\overline{q}}{96}\right)^{\frac{4}{3}}(2r)^{\frac{1}{3}}w^{\frac{2}{3}}$, $\frac{\partial C}{\partial \overline{q}} = \overline{q}^{\frac{1}{3}}\left(\frac{1}{48}\right)^{\frac{4}{3}}r^{\frac{1}{3}}w^{\frac{2}{3}}$, $\frac{\partial C}{\partial \overline{q}}$ increases as output increases.

(iii) $\overline{q} = \frac{p^3 48^4}{rw^2}$

8.17 (i) 42 (ii) 0.75 (iii) -1 (iv) $\frac{\partial y}{\partial w}\cdot\frac{w}{y} = 1$

8.18 (iii) No

Chapter 9

Integration

9.1 $C = 5q + 8q^2 + 32q^3 + 180$

9.2 Consumer surplus = 58.5

9.3 (i) $q = 8$, $p = 160$ (ii) $q = 7$, $p = 165$, The change in consumer surplus is -37.5.

9.4 (i) Consumer surplus = 48 (ii) Consumer surplus = 162 (iii) $\frac{1}{3}$

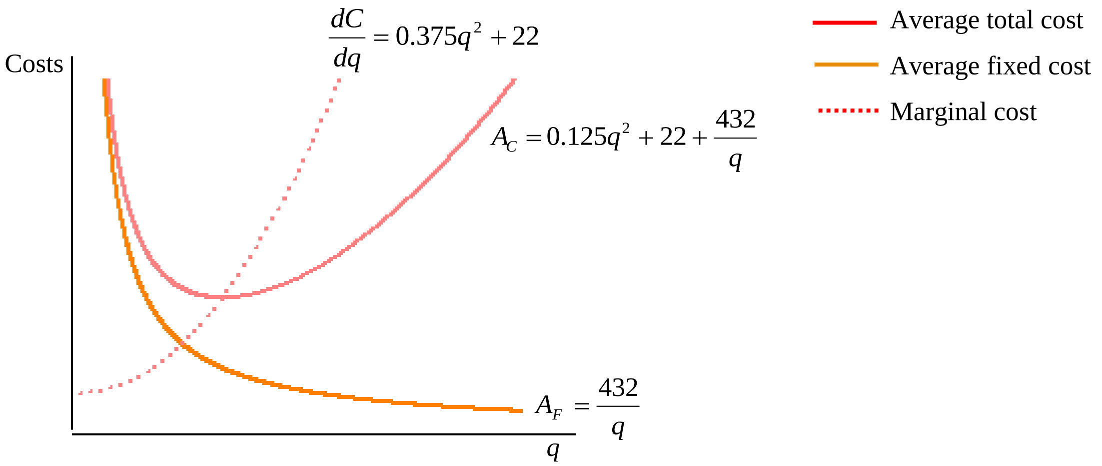
9.5 (i) 48 (ii) 162 (iii) $\frac{1}{3}, \frac{1}{3}$

9.6 (i) 222 (ii) 8,214 (iii) $\frac{dq}{dp} \cdot \frac{p}{q} = -3.86$ (to two decimal places)

9.7 (i) $q = 12$ (ii) Total variable costs are 480 and total fixed costs are 432.

(iii) $A_C = \frac{C}{q} = \frac{0.125q^3 + 22q + 432}{q} = 0.125q^2 + 22 + \frac{432}{q}$

(iv)



9.8 (i) $b = 52$ (ii) $p = 138.4, 460.8$

9.9 £84,000

9.10 $C = \frac{-1{,}000}{(q+2)^2} + 48{,}750$

9.11 (i) $p = \frac{1}{3}\left(\frac{Y^2}{n}\right)^{\frac{1}{3}}$, $q = \frac{(Yn)^{\frac{2}{3}}}{9}$ (ii) *a.* $q = 1{,}024$, $p = 6$

b. $q = 256$, $p = 12$, Consumers pay $\frac{4}{7}$, Producers pay $\frac{4}{7}$

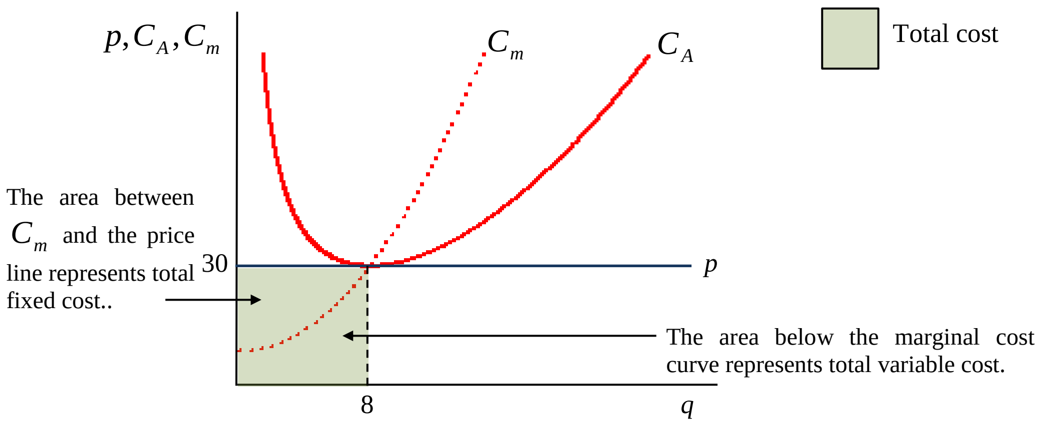
c. $\pi = 3.5$

9.12 (i) $\frac{R}{q} = 4{,}000 - 10q - q^2$ (ii) $\frac{dR}{dq} = -835.2$

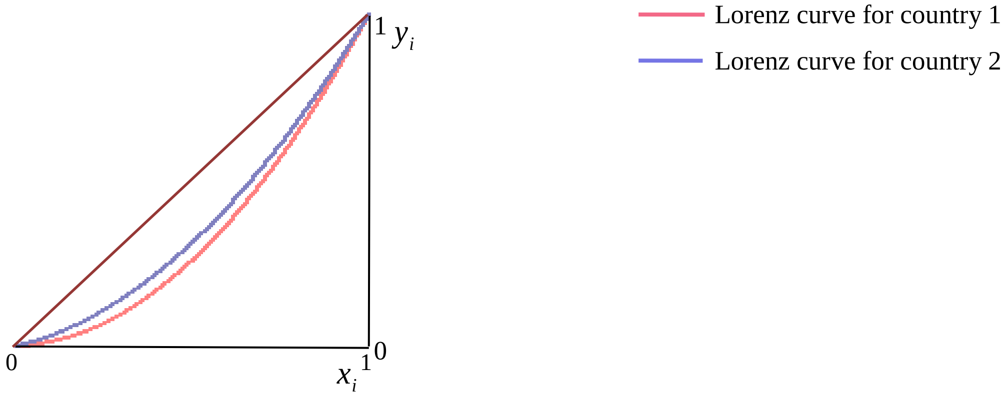
9.13 (i) 564 (ii) 864

9.14 (i) 8 (ii) Total variable costs: 112, Total fixed costs: 128

(iii)



9.15 (i)



The distribution of income is more equal in country 2 than in country 1.

(ii) Country 1: $G_1 = 0.32$, Country 2: $G_2 = 0.25$ (iii) 8%, 32%.

9.16 (i) $C = \dfrac{q^3}{6} + 10q^2 + 2q + 1{,}152$ (ii) 30, 16,848

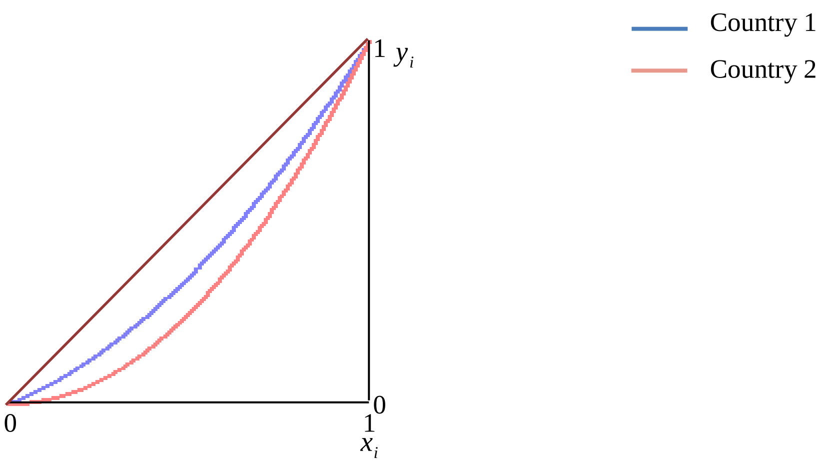
9.17 (i) $q = 324$, $p = 24$ (ii) $\eta = 13\frac{1}{3}$, $\varepsilon = 5\frac{1}{3}$, $p = 9.2$

(iii) Consumer surplus: 388.8, Producer surplus: 972

9.18 (i) 6, 270 (ii) 174, 54

9.19 (i) 12 (ii) 1,089.2 (iii) $\eta = -2.29$

9.20

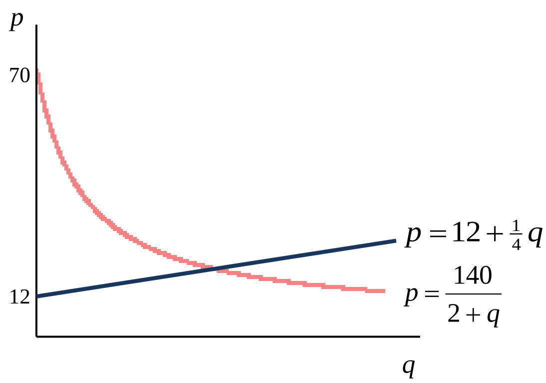


$$G_1 = \frac{1}{5},\ G_2 = \frac{1}{3}$$

9.21 121, 605, 1,331

9.22 1,699

9.23 (i)



(ii) Consumer surplus: 113.32, Producer surplus: 8

(iii) 3.78, Consumers pay: 87.5%, Producers pay: 12.5%

9.24 (i) $q = 256$, $p = 12$ (ii) Consumer surplus: 1,024, Producer surplus: 614.4

(iii) $q = 16$, $R_G = 288$, Consumers pay: $\frac{12}{18} = \frac{2}{3}$, Producers pay: $\frac{6}{18} = \frac{1}{3}$

9.25 The area of the area labelled D and that labelled E is 25.6

9.26 (i) $C = 2d + b\left(\frac{d}{a}\right)^{0.5}$ (ii) d

9.27 (i) $p = 25\frac{1}{2}$, $q = 5$ (ii) Consumer surplus: 6.25, Producer surplus: $58\frac{1}{3}$

(iii) The government raises revenue of 27.2.

Consumption falls by 1 unit, Price increases by £0.5, Consumer surplus falls by 2.25

Net revenue to producers falls by 50.7, Producer surplus falls by $55\frac{2}{3}$

(iv) Consumers pay: 7.4%, producers pay: 92.6%

9.28 (i) $q_1 = 80$, $q_2 = 320$ (ii) $C_1 = q_1^2 + 176$

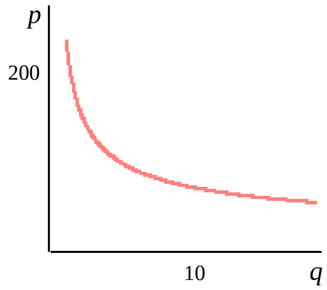
9.29 (i) Change in consumer surplus: $-3{,}221\frac{59}{66}$, change in producer surplus: $3{,}946\frac{2}{3}$

(ii) Net Welfare loss: $675\frac{5}{22}$

9.30 (i) $R = \dfrac{78q}{6+5q}$, $\dfrac{R}{q} = \dfrac{78}{6+5q}$ (ii) $q = 4, \quad p = 3$ (iii) $q = g^{-1}(p) = \dfrac{56}{3p} - \dfrac{4}{3}$

9.31 (i) $G = 0.5$ (ii) $\alpha = 2$ (iii) $G = \frac{1}{3}$, curve 2

9.32 (i) (ii) $\eta = -\dfrac{(4q^{\frac{1}{2}} + 216)}{108}$ (iii) $\eta = -3$ (iv) 5,832



9.33 (ii) 16,128

9.34 (i) $G = \dfrac{1}{2\alpha - 1}$ (ii) $1.3 < \alpha < 1.75$

9.35 (i) $C = 5q + 0.8q^2 + 24$, $R = 20q - 0.7q^2$ (ii) $q = 5$, $\pi = 13.5$, $p = 16.5$.

9.36 (i) $q^* = 15$ (ii) $\dfrac{dC}{dq} = 0.15q^2 - 3q + 16$

(iii) 71.25. This area represents long-run total cost when output is 15.