

(*mathematics*)  *inEconomics*

Mathematical Methods in Economics: Problems and Solutions

Chapter 6

*Problems on Optimising a Function of Two
Variables*

6.1 A monopolist sells output in two different markets and is able to charge a different price in each. The inverse demand function for each market is given below.

$$p_1 = 100 - q_1$$

$$p_2 = 84 - q_2$$

where p_i = price in market i , $i = 1, 2$

q_i = quantity demanded in market i , $i = 1, 2$

The cost function of the monopolist is:

$$C = 600 + 4(q_1 + q_2)$$

where C = total cost

How much should be sold in each market to maximise profit?

6.2 A monopolist is selling a product in two distinct markets and practicing price discrimination. The inverse demand function in each market is given below.

$$\text{Market 1: } p_1 = f(q_1) = 180 - 2q_1$$

$$\text{Market 2: } p_2 = g(q_2) = 132 - 3q_2$$

where p_i = price (£) in market i , $i = 1, 2$

q_i = quantity demanded (units) in market i , $i = 1, 2$

The short-run cost function of the monopolist is given by:

$$C = c(q) = 2q^2 + 20q + 85$$

where C = total cost (£)

q = output (units)

- (i) Determine the quantity the monopolist will sell in each market to maximise profit and the price at which the output will be sold in each market.
- (ii) A change in the preferences of consumers in market 1 results in a shift in the demand curve. To maximise profit the producer now sells 13 units in market 1 at a price of £115.50. Marginal revenue in market 1 is £96. Find the equation of the demand function after preferences change.

6.3 The market demand function for a commodity is given by:

$$q = 500 - 10p$$

where q = quantity demanded

p = price

Two firms supply the market and have the following long-run cost functions:

$$\text{Firm 1: } C_1 = 0.1q_1^2 + 2q_1$$

$$\text{Firm 2: } C_2 = 0.025q_2^2 + 2q_2$$

where C_i = total cost of firm i , $i = 1, 2$

q_i = output of firm i , $i = 1, 2$

- (i) If each firm aims to maximise profit on the assumption that the other firm keeps output constant the output of a firm is given by its reaction function. The reaction functions for these firms take the form:

$$q_1 = 120 - 0.25q_2$$

$$q_2 = 192 - 0.4q_1$$

Determine the quantity that will be supplied to the market, the price at which the output will be sold and the profit each firm will make.

- (ii) If the firms form a cartel and agree to maximise joint profits, determine the quantity supplied to the market and the new market price. What is the profit to producers? What will be necessary for the cartel to be maintained?

- 6.4 A monopolist operating in the services sector is able to sell to different groups of consumers at different prices. The following functions give average revenue in each submarket.

$$p_1 = f(q_1) = 180 - 2q_1$$

$$p_2 = g(q_2) = 48 - \frac{1}{3}q_2$$

where p_i = price in submarket i , $i = 1, 2$

q_i = quantity in submarket i , $i = 1, 2$

The total cost function of the monopolist is given by:

$$C = c(q) = 2,000 + 4q + \frac{1}{2}q^2$$

where C = total cost

q = quantity

- (i) How much will the monopolist sell in each submarket to maximise profit?
(ii) Find the elasticity of demand in each submarket at the profit-maximising output.

- 6.5 A monopolist sells a product in two different markets. The inverse demand function for each market is given by:

$$p_1 = f(q_1) = 70 - 2q_1$$

$$p_2 = g(q_2) = 30 - q_2$$

where p_i = price in market i , $i = 1, 2$

q_i = quantity in market i , $i = 1, 2$

Fixed costs of the firm are £48. Variable costs (V) are given by:

$$V = c(q_1, q_2) = q_1^2 + 2q_1q_2 + q_2^2$$

- (i) Find the quantity the monopolist will sell in each market to maximise profit. What is the level of profit?
(ii) At what price will the product be sold in each market?
(iii) Show that at the levels of output in (i) marginal cost equals marginal revenue in both markets.