(mathematics) √inEconomics

Mathematical Methods in Economics: Problems and Solutions

Chapter 5

Problems on Partial Differentiation

5.1 A firm produces two products and has a total cost function of the form:

$$C = c(q_1, q_2) = 2q_1^3 + 4q_2^{0.5} + 2.4q_1^2 q_2 + 56$$

where C = total cost
q_1 = output of product 1
q_2 = output of product 2

Find the marginal cost of product 1 and the marginal cost of product 2.

5.2 A firm has a production function of the form:

$$q = f(L, K) = 12L^{\frac{1}{2}} K^{\frac{1}{4}}$$

where q = output (units)
L = labour input (units)
K = capital input (units)

(i) Find the marginal product of each input.

(ii) Is the law of diminishing marginal product operating for this production function?

5.3 A utility function takes the form:

$$u = u(q_1, q_2) = \tfrac{3}{4}\ln q_1 + \tfrac{1}{3}\ln q_2$$

where u = index of utility
q_1 = quantity of good 1
q_2 = quantity of good 2

(i) Find the marginal utility of each good.

(ii) Is the law of diminishing marginal utility operating here?

(iii) What conditions are required for the law of diminishing marginal utility to be operating for the utility function $u = f(q_1, q_2)$?

5.4 The short-run total cost (C) function for a multiproduct firm that produces two types of goods is given by the following function where q_i for $i = 1{,}2$ represents the output of good i.

$$C = c(q_1, q_2) = 0.4q_1^3 + 0.36q_2^3 - 6q_1^2 - 4.32q_2^2 + 32q_1 + 20q_2 + 4q_1 q_2 + 60$$

(i) Find the marginal cost function for each good.

(ii) For each good find the levels of output for which marginal cost is decreasing and for which it is increasing

5.5 The long-run total cost function of a firm is given by:

$$C = c(w, r, q) = 10w^{\frac{1}{4}} r^{\frac{1}{2}} q^{\frac{4}{3}}$$

where C = total cost
w = payment per unit of labour input (wage rate)
r = payment per unit of capital input
q = output

(i) Find the long-run marginal cost function for this firm.

(ii) Show that the marginal cost function is a monotonically increasing function.

(iii) Find the elasticity of the total cost function with respect to the wage rate.

5.6 The market demand function for a good is given by:

$$q_a^d = f(p_a, p_b, Y) = \alpha_0 - \alpha_1 p_a + \alpha_2 p_b + \alpha_3 Y$$

where q_a^d = quantity demanded of good *A*

p_a = price of good *A*

p_b = price of a good *B*

Y = household income

All parameters are positive.

Find the elasticities of demand. What can be concluded from these elasticities?

5.7 A consumer has a utility function:

$$u = u(x, y)$$

where u = index of utility

x = quantity of good *X*

y = quantity of good *Y*

The consumer's demand functions are:

$$q_x = \alpha_1 p_y^2 - \beta_1 p_x p_y + \gamma_1 M$$

$$q_y = \alpha_2 p_x^2 - \beta_2 p_x p_y + \gamma_2 M$$

where q_x = quantity demanded of good *X*

q_y = quantity demanded of good *X*

p_x = price of good *X*

p_y = price of a good *Y*

M = income

(i) Find the elasticities of demand for good *X*.

(ii) Determine the condition that p_x and p_y must satisfy for *X* and *Y* to be substitutes.

5.8 The demand functions for the two commodities are:

$$q_1 = 4 + \frac{100}{p_1 + 10} + 2p_2$$

$$q_2 = 6 + 5p_1 + \frac{50}{p_2 + 5}$$

where q_i = quantity demanded of good *i*, i = 1, 2

p_i = price of good *i*,, i = 1, 2

(i) If both goods are supplied free, how much of each good will be demanded?

(ii) Do these goods obey the 'law of demand'?

(iii) What may be concluded about the relationship between these two goods?

(iv) Sketch the demand curve for good 1.

