Mathematical Methods in Economics:

Problems and Solutions

Chapter 4

Problems on Optimising a Function of

One Variable

4.1 A firm operating as a monopoly has a cost function given by:

$$C = c(q) = 82.5 + 2q + 0.1q^2$$

where C = total cost
$\quad\quad q$ = output

and faces the market demand function:

$$p = 20 - 0.2q$$

where p = price

(i) Find the level of output that maximises profit, and hence deduce the monopoly price.

(ii) Show that at the level of output at which profit is maximised, marginal cost is equal to marginal revenue.

(iii) Graph the total revenue and total cost functions on the same diagram. Mark on the diagram the profit-maximising output, the revenue-maximising output, and the outputs at which the firm will break even.

4.2 A monopolist faces an average revenue function of the form:

$$p = 520 - q$$

where p = price
$\quad\quad q$ = quantity

The monopolist's cost function is given by:

$$C = 1,470 + 24q + q^2$$

where C = total cost

What level of output should the monopolist produce in order to maximise profit? Show that marginal revenue is equal to marginal cost at this level of output.

4.3 Consumers' demand for a certain commodity is given by:

$$q = 16,200 - 150p$$

where q = quantity demanded (units) per month
$\quad\quad p$ = price (£)

Express consumers' total monthly expenditure on this commodity as a function of p and determine the price that will result in maximum expenditure.

4.4 A perfectly competitive firm has a short-run total cost function of the form:

$$C = q^3 - 30q^2 + 280q + 600$$

where C = total cost
$\quad\quad q$ = output

To what level would price have to fall for the firm to stop operating in the short-run?

4.5   A firm has a total cost function given by:

$$C = c(x) = x^2 + 7x + 144$$

where x = output (units)
      C = total cost (£000's)

(i)   Find the output at which average total cost is minimised.

(ii) If the firm is limited in the resources that it can obtain so that the maximum output that it can produce is 8 units, what output should it produce in order to minimise average cost?

4.6   A monopolist is constrained by the availability of raw materials to producing 120 units of output. The average revenue function the monopolist faces is:

$$p = f(x) = 538 - x$$

and the cost function is:

$$C = g(x) = 100 + 6x + x^2$$

where C = total cost
      p = price
      x = output (units)

Determine the output that the monopolist should produce to maximise profit.

4.7   The total costs of a firm are given by the function:

$$C = c(q) = \tfrac{1}{12}q^3 - \tfrac{5}{2}q^2 + 36q + 85$$

where C = total cost (£)
      q = output (units)

The total revenue function is:

$$R = 27q$$

where R = total revenue (£)

(i)   Could this firm be operating in a perfectly competitive market?

(ii) Determine the level of output the firm should produce if its objective is to maximise profit. What is the maximum level of profit?

4.8   A monopolist is faced by the following demand function:

$$p = 144 - 8q$$

where p = price
      q = quantity demanded

Average cost (A) is given by:

$$A = q$$

Find the level of output that maximises profit, the level of profit, and total revenue and total cost, when profit is being maximised. Represent the average and marginal functions graphically and indicate on the diagram the output at which profit is maximised.