Mathematical Methods in Economics: Problems and Solutions

Chapter 3

Problems on Differentiating a Function of One Variable

3.1 Total costs of a firm are given by the function:

$$C = c(q) = 15 + q^2$$

where C = total cost
q = output

Find the marginal cost function. Draw the total cost curve and marginal cost curve on separate diagrams.

3.2 The inverse market demand function for a product a firm produces is given by the function:

$$p = f(x) = 6 - 0.8x$$

where p = price
x = quantity demanded

Find the total revenue function and the marginal revenue function. Represent these functions graphically.

3.3 A consumption function takes the form:

$$C = c(Y) = 4 + 0.6Y + 2Y^{0.5}$$

where C = consumption expenditure (£ million)
Y = disposable income (£ million)

Find the marginal propensity to consume. How does it change as income increases?

3.4 A firm has a total revenue function of the form:

$$R = f(q) = 10\sqrt{2q + 5}$$

where R = total revenue
q = output

Find marginal revenue when 2 items are sold.

3.5 A firm has a short-run production function of the form:

$$Q = q(L) = 60L^2 - L^3$$

where Q = output
L = labour input

(i) Find the marginal product of labour function.

(ii) Over what range of labour input is the law of diminishing marginal product operating? Represent $q(L)$, $\dfrac{dQ}{dL}$ and $\dfrac{d^2Q}{dL^2}$ graphically.

3.6 Total revenue for a firm is given by the function:

$$R(q) = 3q + \frac{224q}{q+4}$$

where q = output

Find marginal revenue when price is 10.

3.7 Find the derivative of the average fixed cost function $a(q)$, where q represents output, for a firm with total fixed costs equal to F.

3.8 For the total cost function:

$$C(x) = \frac{x^3}{10} - 47x^2 + 8{,}000x + 58{,}500$$

where $C(x)$ = total cost
x = output

Find the average and marginal cost functions. Find average and marginal cost when $x = 180$.

3.9 A firm has a short-run production function of the form:

$$Q = q(L) = aL + bL^2$$

where Q = output
L = labour input
$a, b > 0$

(i) Find the marginal product of labour.

(ii) Is the law of diminishing marginal product operating?

3.10 A manufacturer has obtained the following information on the costs of production:

Output (q)	5	10	15
Total cost	20	65	160

(i) Find the equation of the cost function on the assumption that this function is quadratic.

(ii) *a*. What is the cost of producing 20 units of output?

b. What are the fixed costs?

(iii) If the total revenue function for this manufacturer is given by:

$$R = f(q) = 20q$$

find the levels of output at which total revenue is equal to total cost.

(iv) Find the average fixed cost function, the average variable cost function and the marginal cost function.

