

( *mathematics* )  *inEconomics*

# **Mathematical Methods in Economics: Problems and Solutions**

## Chapter 1

### *Problems on Equations and Functions I*

- 1.1 The demand function for a good is defined by the following equation where  $q_d$  represents quantity demanded (units) and  $p$  represents market price (€).

$$q_d = 48 - 4p$$

- (i) How many units of this good will consumers demand at each price given below?

$p$	0	3	6	8	12
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- (ii) What would consumers' total expenditure be at each price in (i)?  
 (iii) Find  $p$  in terms of  $q_d$  and represent this equation graphically. What term do economists use to describe the graph of this equation?

- 1.2 The supply function for a good is defined by the following equation where  $q_s$  represents quantity supplied, and  $p$ , market price.

$$q_s = f(p) = a + bp$$

- (i) What information is given by the notation  $f(p)$ ? Why is it not necessary to include this notation when defining the demand function in question 1.1?  
 (ii) Find  $f(0)$ ,  $f(4)$  and  $f(d)$ .  
 (iii) Find  $p$  in terms of  $q_s$ .  
 (iv) Draw the graph of the equation in (iii) on the assumptions that  $a < 0$  and  $b > 0$ . What term do economists use to describe this graph?

- 1.3 A consumption function takes the following form where  $C$  represents consumers' expenditure and  $Y$  represents national income.

$$C = c(Y) = 100 + 0.8Y$$

- (i) How much will consumers spend at each level of national income given in the table?

$Y$	200	201	250	275	336	480
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- (ii) Find  $c(0)$ ,  $c(120)$ ,  $c(345)$  and  $c(m)$ . What does  $c(0)$  represent in this model?  
 (iii) Saving ( $S$ ) by consumers is defined as  $S = Y - C$ . The savings function expresses  $S$  in terms of  $Y$ . Find this function.  
 (iv) Draw the graph of the consumption function and savings function on the same diagram.  
 (v) Find  $\frac{\Delta C}{\Delta Y}$  for the sequence of  $Y$  values given in (i). What does this difference quotient represent?

1.4 A short-run production function takes the form:

$$q = q(L) = 6L^{\frac{1}{2}}$$

where  $q$  = output  
 $L$  = labour input

- (i) How much output will be produced by a firm that employs each of the following quantities of labour?

Labour input					
0	1	4	9	25	100

- (ii) Find  $\frac{\Delta q}{\Delta L}$  for the sequence of values of  $L$  given in (i). What does this difference quotient represent?
- (iii) Represent  $q(L)$  graphically. What name is used in economics for this graph?

1.5 A firm has a long-run production function that takes the following form where  $q$  represents output,  $L$  represents labour input and  $K$  represents capital input.

$$q = f(L, K) = 20L^{0.5} K^{0.5}$$

- (i) Find the level of output that will result from the use of each of the following combinations of inputs.

$L$	$K$
0	4
1	4
4	4
9	4
16	4
18	8
36	9
27	12

- (ii) Find  $f(0,0)$ ,  $f(7,28)$ ,  $f(a,b)$  and  $f(L_0, K_0)$ .
- (iii) Use information obtained in (i) to draw the graph of the relationship between  $q$  and  $L$  when 4 units of capital are used. What term is used in economics to describe this graph?
- (iv) Find the expression giving  $L$  in terms of  $K$  when  $q = 120$ .
- (v) Find six different combinations of capital and labour that the firm could employ to produce 120 units of output *efficiently*.
- (vi) Use the values obtained in (v) to draw the graph of the relationship between  $L$  and  $K$  when  $q = 120$ . What term is used in economics to describe this graph?